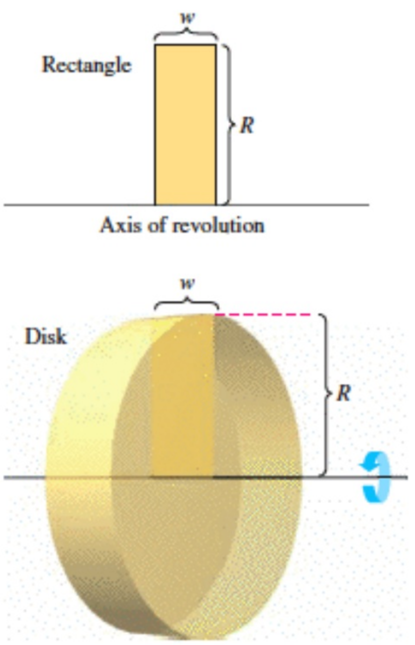
7.2 Volume: The Disk Method

- Find the volume of a solid of revolution using the disk method.
- Find the volume of a solid of revolution using the washer method.



Volume of a disk: $\pi R^2 w$

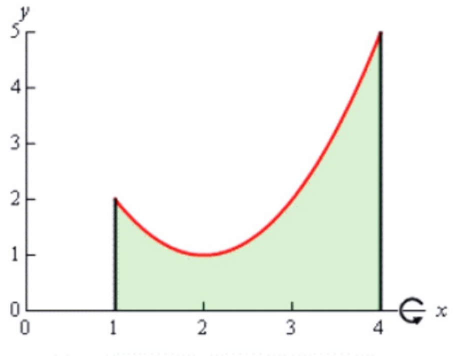
$$A_{circle} = \pi r^2$$

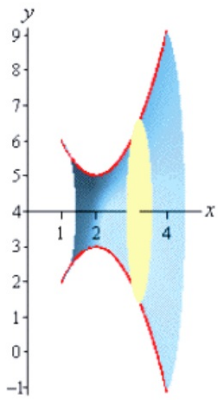
$$V_{cylinder} = \pi r^2 h$$

Volume by Revolution (Disk Method)

$$A = \pi\left(\begin{pmatrix}\text{outer}\\ \text{radius}\end{pmatrix}^2 - \begin{pmatrix}\text{inner}\\ \text{radius}\end{pmatrix}^2\right)$$

Example 1 Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 4x + 5$, $x = 1$, $x = 4$, and the x-axis about the x-axis.





$y=x^2 - 4x + 5$
$x = 1, x = 4,$
rotate about the x-axis

#2 Find the volume of the solid that is formed by $y = x^2 - 9$, $y = 4$; rotated about $y = 4$.

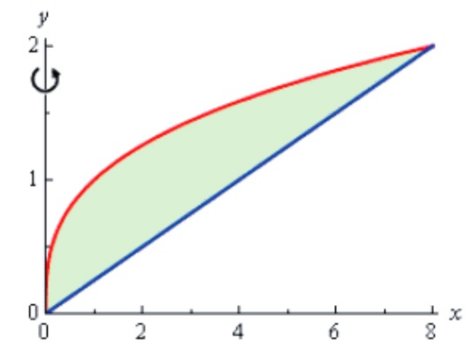
#3 Region: $y = x^2$, $x = 0$, $y = 4$ (1st quad) rotated about y-axis

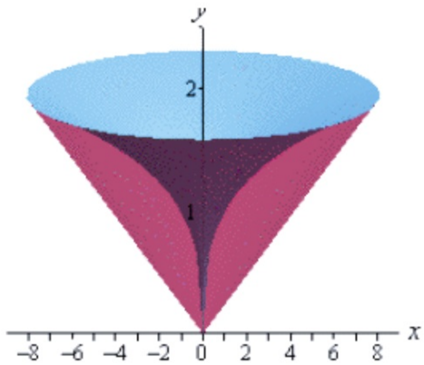
Volume by Revolution (Washer Method)

$$V = \pi \int_{a}^{b} \left[R(x)\right]^2 - \left[r(x)\right]^2 dx$$

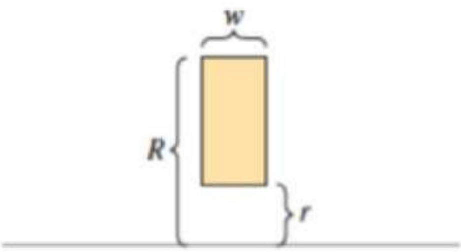
$$V = \pi \int_{\square}^{b} \left[(outer - axis)^2 - (inner - axis)^2 \right] dx$$

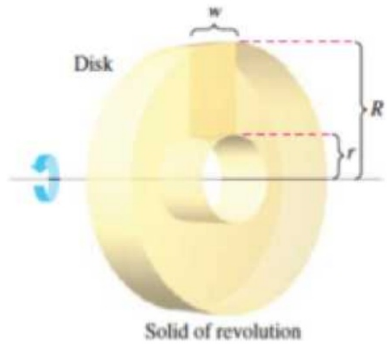
Example 2 Determine the volume of the solid obtained by rotating the portion of the region bounded by $y = \sqrt[3]{x}$ and $y = \frac{x}{4}$ that lies in the first quadrant about the y-axis.





Washer Method





$$A_{\text{washer}} = \pi R^2 - \pi r^2 = \pi\left(R^2 - r^2\right)$$

$$V_{\text{cylinder}} = \pi R^2 h - \pi r^2 h = \pi\left(R^2 - r^2\right)h$$

Washer Method

Horizontal Axis of Revolution

$$V = \pi \int_a^b \left([R(x)]^2 - [r(x)]^2 \right) dx$$

Vertical Axis of Revolution

$$V = \pi \int_a^b \left([R(y)]^2 - [r(y)]^2 \right) dy$$

#4 $y = \sqrt[3]{x}$ $y = \frac{x}{4}$ rotate : y-axis

1st Quad.

#5 *Find the volume of the solid obtained by rotating the region bounded by the curve* $y = x^2$ *and* $y = \sqrt{x}$ *about the line* $y = -1$.

#6

Find the volume of the solid obtained by rotating the region bounded by $x = y^2 - 2$ *and* $y = x$ *about the line* x=4

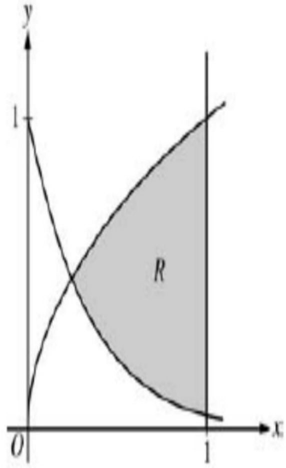
AP Practice Question

Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.



Answers to AP Question

Point of intersection

$e^{-3x} = \sqrt{x}$ at $(T, S) = (0.238734, 0.488604)$

(a) Area $= \int_T^1 \left(\sqrt{x} - e^{-3x}\right) dx$

$= 0.442$ or 0.443

(b) Volume $= \pi \int_T^1 \left(\left(1 - e^{-3x}\right)^2 - \left(1 - \sqrt{x}\right)^2\right) dx$

$= 0.453\pi$ or 1.423 or 1.424

(c) Length $= \sqrt{x} - e^{-3x}$

Height $= 5\left(\sqrt{x} - e^{-3x}\right)$

Volume $= \int_T^1 5\left(\sqrt{x} - e^{-3x}\right)^2 dx = 1.554$

1: Correct limits in an integral in (a), (b), or (c)

2: { 1 : integrand; 1 : answer }

3: { 2 : integrand
- < −1 > reversal
- < −1 > error with constant
- < −1 > omits 1 in one radius
- < −2 > other errors

1 : answer }

3: { 2 : integrand
- < −1 > incorrect but has $\sqrt{x} - e^{-3x}$ as a factor

1 : answer }