5.1/5.3 Slope Fields & Solving Differential Equations - Notes

- What is a Slope Field? ____

____

- Sketching a Slope Field

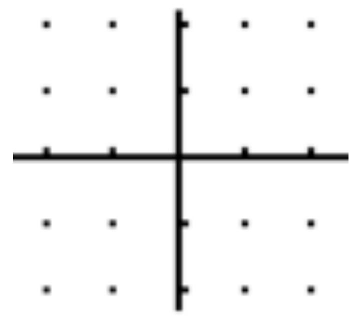
1. ____
2. ____

Ex 1: Sketch each slope field.

a) $\frac{dy}{dx} = 2$

b) $\frac{dy}{dx} = x - 1$

c) $\frac{dy}{dx} = -3y$



d) $\frac{dy}{dx} = 2x + y$

e) $\frac{dy}{dx} = y + xy$

Ex 2: Consider the differential equation given by $\frac{dy}{dx} = \frac{x}{y}$.

a) Sketch a slope field.

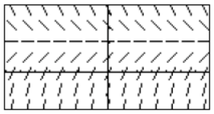
b) On the slope field above, sketch a solution curve that passes through the point (0, 1).

c) On the slope field above, sketch a solution curve that passes through the point (0, -1).

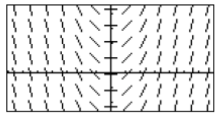
- Matching Slope Fields with Differential Equations:____

Ex 3: Match each differential equation with the slope field.

(A)



(B)



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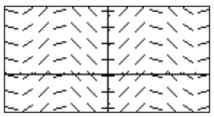

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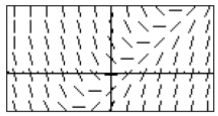


I. $\dfrac{dy}{dx} = \sin x$

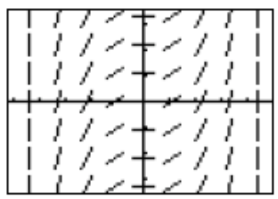
II. $\dfrac{dy}{dx} = x - y$

III. $\dfrac{dy}{dx} = 2 - y$

IV. $\dfrac{dy}{dx} = x$

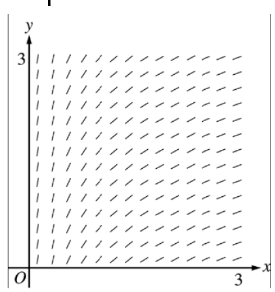
- Matching Slope Fields with Equations:____

Ex 4: The slope field for a certain differential equation is shown below. Which of the following could be a particular solution to the differential equation?



(a) $y = \sin x$ (b) $y = \cos x$ (c) $y = x^2$ (d) $y = \dfrac{1}{6}x^3$ (e) $y = \ln x$

Ex 5: The slope field for a certain differential equation is shown below. Which of the following could be a particular solution to the differential equation?



(a) $y = x^2$ (b) $y = e^x$ (c) $y = e^{-x}$ (d) $y = \cos x$ (e) $y = \ln x$

Ex 6: Verify the solution of the differential equation.

a)

Solution	Differential Equation
$y = e^{-2x}$	$3y' + 5y = -e^{-2x}$

b)

Solution	Differential Equation
$y = 3\cos x + \sin x$	$y'' + y' = 0$

Two Types of Solutions to Differential Equations

1. ______________________________

2. ______________________________

Ex 7: Find the general solution.

a) $y' = \frac{2x}{y}$	b) $y' = 3y$

Ex 8: Find the particular solution.

a) $y' = 7y$, (10, 1)	b) $y' = \frac{x}{y}$, (0, -1)
c) $y' = \frac{y}{x^2}$, (1, 3)	d) $y\sqrt{1-x^2}\,y' - x\sqrt{1-y^2} = 0$, (0, 1)

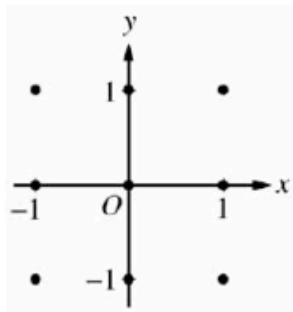
Ex 9: The rate of change of y with respect to x is proportional to the difference between x and 4. Write a differential equation.

Ex 10: The rate of change of y with respect to x varies directly with the square of y. Write a differential equation.

Ex 11:

Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated. **(Note: Use the axes provided in the exam booklet.)**



(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

Ex 12:

Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2+1}{2y}$.

(a) Find the slope of the graph of f at the point where $x = 1$.

(b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.

(c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2+1}{2y}$ with the initial condition $f(1) = 4$.

(d) Use your solution from part (c) to find $f(1.2)$.

Ex 13:

Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

(b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

Ex 14:

Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the test booklet.)**

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.