4.6 The Natural Logarithmic Function: Integration

Review:

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

$$\frac{d}{dx}[\ln|x|] = \frac{1}{x}$$

$$\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot u'$$

THEOREM 4.19 Log Rule for Integration

Let u be a differentiable function of x.

1. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$

2. $\displaystyle\int \frac{1}{u}\,du = \ln|u| + C$

ex: Integrate.

a) $\int \frac{6}{x} dx$

$6 \ln|x| + C$

ex: Integrate.

b) $\int \frac{1}{(3x+5)}dx = \frac{1}{3}\int \frac{1}{u}du$

$u = 3x+5$

$du = 3du$

$\frac{1}{3}\ln|3x+5| + c$

ex: Integrate.

c) $\int \frac{2x}{x^2+6}dx = \int \frac{1}{u}du$

$u = x^2 + 6$ $\quad = \ln|u| + C$

$du = 2x\,\cancel{dx}$ $\quad = \ln|x^2+6| + C$

On your own...

ex: Integrate.

$\int \frac{\sin x}{\cos x} dx$ $u = \cos x$ $du = -\sin x$

d) $\int \frac{\sec^2 x}{\tan x} dx$

g) $\int \frac{x}{\sqrt{9-x^2}} dx$

e) $\int \frac{\sec^2 x}{\tan^2 x} dx$

$\int \frac{\sec^2 x}{(\tan x)^2} dx$

h) $\int \frac{1}{x \ln x} dx$

$u = \ln x$

$du = \frac{1}{x} dx$

$\int \frac{1}{u} du = \ln|\ln x| + C$

f) $\int \tan x \, dx$ $\int \frac{\sin x}{\cos x} dx$

i) $\int \frac{x^3 + 4x^2 - 2x + 1}{x^2} dx$

ex: Integrate.

d) $\int \frac{\sec^2 x}{\tan x}\,dx$

ex: Integrate.

e) $\int \frac{\sec^2 x}{\tan^2 x}\,dx$

ex: Integrate.

f) $\int \tan x \, dx$

ex: Integrate.

g) $\int \frac{x}{\sqrt{9-x^2}}dx$

ex: Integrate.

h) $\int \frac{1}{x \ln x} dx$

ex: Integrate.

i) $\displaystyle\int \frac{x^3+4x^2-2x+1}{x^2}\,dx$

ex: Integrate.

When the degree of the numerator is $\geq$ degree of the denominator, divide!

j) $\cancel{\int \frac{x^2+x+1}{x^2+1}\,dx}$

$$\int \frac{x^2+7x+3}{x+1}\,dx$$

$$\begin{array}{r|rrr} -1 & 1 & 7 & 3 \\ & & -1 & -6 \\ \hline & 1 & 6 & -3 \end{array}$$

$$\int\left(x+6-\frac{3}{x+1}\right)dx = \frac{x^2}{2}+6x-3\ln|x+1|+C$$

Trigonometric Antiderivatives

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \tan x\,dx = -\ln|\cos x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

ex: Integrate.

$$\int \frac{\ln x}{x}\,dx$$

a) $\int \tan 5x\,dx$

$$\int \frac{\sin 5x}{\cos 5x}\,dx = -\frac{1}{5}\int \frac{1}{u}\,du$$

$u = \cos 5x$

$du = -5\sin 5x\,dx$

$$-\frac{1}{5}\ln|\cos 5x| + C$$

ex: Integrate.

b) $\int_{\pi/4}^{\pi/2} \sqrt{\csc^2 x - 1}\, dx$

$\sqrt{\cot^2 x} = \cot x$

$\int_{\pi/4}^{\pi/2} \cot x\, dx = \ln|\sin x| \Big]_{\pi/4}^{\pi/2}$

$\ln|1| - \ln\frac{\sqrt{2}}{2}$

$-\ln\frac{\sqrt{2}}{2} = \ln\left(\frac{2}{\sqrt{2}}\right)$

$\ln\sqrt{2} = \frac{\ln 2}{2}$

ex:

Evaluate $\int_0^{\frac{\pi}{4}} \frac{2e^{\tan x} + 5}{\cos^2 x} dx$

(A) $2e+3$ (B) $2e$ (C) $2e-3$ (D) e (E) $e+5$

ex:

Evaluate $\int_{e}^{e^4} \frac{5}{x\sqrt{\ln x}}\,dx$

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

FR 7

A particle starts at the point $(5,0)$ at $t=0$ and moves along the x-axis in such a way that at time $t>0$ its velocity $v(t)$ is given by $v(t)=\dfrac{t}{1+t^2}$.

(a) Determine the maximum velocity attained by the particle. Justify your answer.

(b) Determine the position of the particle at $t=6$.

(c) Find the limiting value of the velocity as t increases without bound.

FR 21

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ $\left(\text{ft}/\text{sec}^2\right)$	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\, dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\, dt$ using a trapezoidal approximation with the three subintervals determined by the table.

(b) Using appropriate units, explain the meaning of $\int_{0}^{30} a(t)\, dt$ in terms of the car's motion. Find the exact value of $\int_{0}^{30} a(t)\, dt$.