4.5 U-Substitution - cont.

ex: Evaluate.

a) $\int_0^1 x\left(x^2+1\right)^3 dx = \frac{1}{2}\int u^3 du$ $\frac{15}{8}$

$u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$= \frac{1}{2}\cdot\frac{u^4}{4} + C$

$= \frac{1}{8}\left(x^2+1\right)^4 + C\Big]_0^1$

$\frac{1}{8}\left[(16+\cancel{C}) - (1+\cancel{C})\right]$

ALTERNATE WAY

ex: Evaluate.

a) $\int_0^1 x\left(x^2+1\right)^3 dx = \frac{1}{2}\int_1^2 u^3 du =$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}\cdot\frac{u^4}{4}\Big]_1^2$

$\frac{1}{8}(16-1) = \frac{15}{8}$

ex: Evaluate.

b) $\int_{0}^{\pi/2} \cos\left(\frac{2x}{3}\right) dx$

ALTERNATE WAY

ex: Evaluate.

b) $\int_0^{\pi/2} \cos\left(\frac{2x}{3}\right)dx$

$u = \frac{2x}{3}$

$du = \frac{2}{3}dx$

$\frac{3}{2}\int_0^{\pi/3} \cos u \, du$

$\frac{3}{2}\sin u \Big]_0^{\pi/3}$

$\frac{3}{2}\left(\frac{\sqrt{3}}{2} - 0\right) = \frac{3\sqrt{3}}{4}$

ex: Evaluate.

c) $\int_0^1 \frac{e^{\tan^{-1} x}}{1+x^2} dx$ $= \int_0^{\pi/4} e^u \, du$

$u = \tan^{-1} x$

$du = \frac{1}{x^2+1} dx$

$= e^u \Big]_0^{\pi/4}$

$= e^{\pi/4} - 1$

ex: Rewrite the definite integral in terms of u.

Do not evaluate.

a) Let $u = 3x - 2$; $\int_0^1 (3x-2)^3 \, dx$

$du = 3dx$

$$\frac{1}{3}\int_{-2}^{1} u^3 \, du$$

ex: Rewrite the definite integral in terms of u.

b) Let $u = 2x + 3$; $\int_{-1}^{1} \frac{1}{2x+3} dx$

$$\frac{1}{2}\int_{1}^{5} \frac{1}{u} du$$

ex: Rewrite the definite integral in terms of u.

d) Let $u = 4 - x^2$; $\int_0^{\sqrt{3}} \frac{x\,dx}{\sqrt{4-x^2}}$

$$-\frac{1}{2}\int_4^1 u^{-1/2}\,du = \frac{1}{2}\int_1^4 u^{-1/2}\,du$$

ex:

Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ is equal to which of the following?

(A) $2\int_1^{16} e^u\,du$ (B) $2\int_1^{4} e^u\,du$ (C) $2\int_1^{2} e^u\,du$ (D) $\cancel{\frac{1}{2}\int_1^{2} e^u\,du}$ (E) $\cancel{\int_1^{4} e^u\,du}$

$$u = \sqrt{x}$$

$$du = \frac{1}{2\sqrt{x}}\,dx$$

$$2\int_1^2 e^u\,du$$

ex:

Evaluate the definite integral: $\int_0^1 \left(1+e^{-x}\right)^2 dx$.

(A) $\frac{3}{2}-2e+\frac{1}{2}e^2$ (B) $\frac{7}{2}+\frac{2}{e}+\frac{1}{2e^2}$ (C) $\frac{3}{2}-2e-\frac{1}{2}e^2$ (D) $\frac{3}{2}+2e+\frac{1}{2}e^2$

(E) $\frac{7}{2}-\frac{2}{e}-\frac{1}{2e^2}$

$$\int_0^1 \left(1+2e^{-x}+e^{-2x}\right)dx$$

FR 18

The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

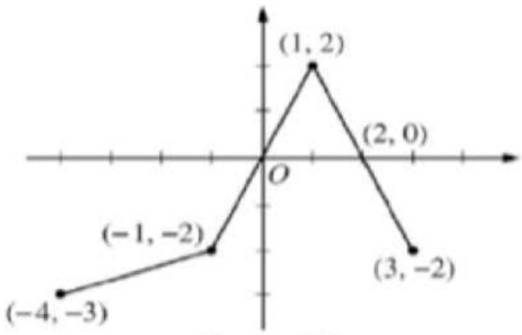
(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

FR 15

The graph of the function f above consists of three line segments.



Graph of f

(a) Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$.

For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

(b) For the function g defined in part (a), find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

(c) Let h be the function given by $h(x) = \int_{x}^{3} f(t)\,dt$. Find all values of x in the closed interval $-4 \leq x \leq 3$ for which $h(x) = 0$.

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

FR 19

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

(a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

(b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

(c) The scientist proposes the function f, given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

(d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water must be diverted?

FR 10

The acceleration of a particle moving along a straight line is given by $a = 10e^{2t}$.

(a) Write an expression for the velocity v, in terms of time t, if $v = 5$ when $t = 0$.

(b) During the time that the velocity increases from 5 to 15, how far does the particle travel?

(c) Write an expression for the position s, in terms of time t, of the particle if $s = 0$ when $t = 0$.