4.5 U-Substitution

U-Substitution is an integration technique used when an integrand involves a composite function.

$$\boxed{f(g(x))}$$

Review:

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$$

THEOREM 4.15 Antidifferentiation of a Composite Function

Let g be a function whose range is an interval I, and let f be a function that is continuous on I. If g is differentiable on its domain and F is an antiderivative of f on I, then

$$\int f'(g(x))g'(x)\,dx = f(g(x)) + C$$

Evaluate.

a) $\int 2x\left(\underline{1+x^2}\right)^5 dx \quad = \quad \dfrac{\left(1+x^2\right)^6}{6} + C$

Check:

$\dfrac{\cancel{6}}{\cancel{6}} \cdot \left(1+x^2\right)^5 \cdot 2x$

Evaluate.

a) $\int 2x(1+x^2)^5 dx$

$u = 1 + x^2$

$du = 2x\,dx$

$\int u^5 du$

$\frac{1}{6}u^6 + C$

$\frac{1}{6}(1+x^2)^6 + C$

ex: Evaluate.

b)

$$\int x(3x^2-5)^{12}\,dx$$

$u = 3x^2 - 5$

$du = 6x\,dx$

$\frac{du}{6} = x\,dx$

$$\int u^{12}\frac{du}{6}$$

$$\frac{1}{6}\int u^{12}\,du$$

$$\frac{1}{6}\cdot\frac{u^{13}}{13}+C$$

$$\frac{(3x^2-5)^{13}}{78}+C$$

ex: Evaluate.

c) $\int 5\sin(7x)\,dx = 5\int \sin(7x)\,dx$

$u = 7x$

$du = 7dx$

$\frac{du}{7} = dx$

$5\int \sin(u)\frac{du}{7}$

$\frac{5}{7}\int \sin u\,du$

$-\frac{5}{7}\cos(7x) + C$

ex: Evaluate.

$\frac{d}{dx}\left(\frac{1}{3}\tan^3 x\right) = 1(\tan^2 x)\sec^2 x$

d) $\int \tan^2 x \sec^2 x \, dx$

$\int (\tan x)^2 \sec^2 x \, dx = \int u^2 \, du$

$u = \tan x$

$du = \sec^2 x \, dx$

$\frac{u^3}{3} + C$

$\frac{\tan^3 x}{3} + C$

ex: Evaluate.

e)

$$\int (x+1)e^{x^2+2x}\,dx = \frac{1}{2}\int e^u\,du$$

$$\frac{1}{2}e^u + C$$

$$\frac{1}{2}e^{x^2+2x} + C$$

$u = x^2 + 2x$

$du = (2x+2)dx$

$du = 2(x+1)dx$

$\frac{du}{2} = (x+1)dx$

ex: Evaluate.

f) $\displaystyle\int \frac{e^{2x}+2e^{x}+5}{e^{x}}\,dx$ $= \displaystyle\int (e^{x}+2+5e^{-x})\,dx$

$e^{x}+2x+5\displaystyle\int e^{-x}\,dx$

$e^{x}+2x-5e^{-x}+C$

ex: Evaluate.

g)

$$\int (\cos 2x - \sin 3x)\, dx$$

$$\frac{1}{2}\sin 2x + \frac{1}{3}\cos 3x + C$$

ex: Evaluate.

h)

$$\int \frac{x}{\sqrt[3]{1-2x^2}}\,dx = -\frac{1}{4}\int u^{-1/3}\,du$$

$$u = 1-2x^2$$

$$du = -4x\,dx$$

$$\frac{du}{-4} = x\,dx$$

$$= -\frac{1}{4}\cdot\frac{u^{2/3}}{2/3} + C$$

$$-\frac{3}{8}(1-2x^2)^{2/3} + C$$

ex: Evaluate.

i)

$$\int \cos^4(x)\sin(x)\,dx$$

ex:

$\int e^x \cos\left(e^x + 1\right) dx =$

(A) $\sin\left(e^x + 1\right) + C$

(B) $e^x \sin\left(e^x + 1\right) + C$

(C) $e^x \sin\left(e^x + x\right) + C$

(D) $\frac{1}{2}\cos^2\left(e^x + 1\right) + C$

FR 1

A particle moves along the x-axis in such a way that its acceleration at time t for $t \geq 0$ is given by $a(t) = 4\cos(2t)$. At time $t = 0$, the velocity of the particle is $v(0) = 1$ and its position is $x(0) = 0$.

(a) Write an equation for the velocity $v(t)$ of the particle.

(b) Write an equation for the position $x(t)$ of the particle.

(c) For what values of t, $0 \leq t \leq \pi$, is the particle at rest?