Make sure you are using the revised syllabus for the rest of this chapter.

More techniques of integration are on the horizon. Keep practicing!! To stay on contract for 4th quarter, you need at least a C this quarter!

15.) $\int (5x \sqrt[3]{1-x^2})\,dx$ $\quad 5\int u^{1/3} \frac{du}{-2}$

$u = 1 - x^2$

$du = -2x\,dx$

$\frac{du}{-2} = x\,dx$

$-\frac{5}{2}\int u^{1/3}\,du$

$-\frac{5}{2}\frac{u^{4/3}}{4/3} + C$

$-\frac{15}{8}(1-x^2)^{4/3} + C$

47.) $\int e^{\sin \pi x} \cos \pi x \, dx$

$u = \sin \pi x$

$du = \cancel{\pi} \cos \pi x \, dx$

$\frac{du}{\pi}$

$\int e^{u} \frac{du}{\pi}$

$\frac{1}{\pi} \int e^{u} du$

$\frac{1}{\pi} e^{u} + C$

$\frac{1}{\pi} e^{\sin \pi x} + C$

$\int x^2 dx$

$\frac{x^3}{3} + C$

45.) $\int \frac{5-e^x}{e^{2x}}\,dx = \int (5e^{-2x} - e^{-x})\,dx$

$-\frac{5}{2}e^{-2x} + e^{-x} + C$

check:

$5e^{-2x} - e^{-x}$

4.5 U-Substitution - cont.

ex: Evaluate.

a) $\int_0^1 x\left(x^2+1\right)^3 dx$ $= \frac{1}{2}\int u^3 du = \frac{1}{2}\cdot\frac{u^4}{4}$

$u = x^2+1$

$du = 2x\,dx$

$= \left.\frac{(x^2+1)^4}{8}\right|_0^1$

$(2) - \left(\frac{1}{8}\right)$

$1\frac{7}{8}$

ALTERNATE WAY

Change of variable

ex: Evaluate.

a) $\int_0^1 x\left(x^2+1\right)^3 dx = \frac{1}{2}\int_1^2 u^3\,du = \frac{u^4}{8}\Big|_1^2$

$u = x^2+1$

$du = 2x\,dx$

$= 2 - \frac{1}{8}$

$\frac{15}{8}$

ex: Evaluate.

b) $\int_0^{\pi/2} \cos\left(\frac{2x}{3}\right)dx$ $= \frac{3}{2}\int_0^{\pi/3} \cos u\,du$

$u = \frac{2x}{3}$

$du = \frac{2}{3}dx$

$\frac{3}{2}\sin u \Big|_0^{\pi/3}$

$\frac{3}{2}\left(\frac{\sqrt{3}}{2} - 0\right)$

$\frac{3\sqrt{3}}{4}$

ALTERNATE WAY

ex: Evaluate.

b) $\int_{0}^{\pi/2} \cos\left(\frac{2x}{3}\right)dx$

ex: Rewrite the definite integral in terms of u.

a) Let $\underline{\underline{u = 3x - 2}}$; $\int_{0}^{1} (3x-2)^3\,dx$

$$\frac{1}{3}\int_{-2}^{1} u^3\,du$$

ex: Rewrite the definite integral in terms of u.

b) Let $u = 2x + 3$; $\int_{-1}^{1} \frac{1}{2x+3} dx$

$$\frac{1}{2}\int_{1}^{5} \frac{1}{u} du = \frac{1}{2}\ln|u| \Big|_{1}^{5}$$

$$\frac{1}{2}(\ln 5 - 0)$$

ex: Rewrite the definite integral in terms of u.

d) Let $u = 4 - x^2$; $\int_0^{\sqrt{3}} \frac{x\,dx}{\sqrt{4-x^2}}$

$du = -2x\,dx$

$$-\frac{1}{2}\int_4^1 \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int_1^4 \frac{1}{\sqrt{u}}\,du$$

ex:

Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$ is equal to which of the following?

(A) $\cancel{2\int_1^{16} e^u\, du}$ (B) $\cancel{2\int_1^{4} e^u\, du}$ (C) $2\int_1^{2} e^u\, du$ (D) $\cancel{\frac{1}{2}\int_1^{2} e^u\, du}$ (E) $\int_1^{4} e^u\, du$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx$

$2du = \frac{dx}{\sqrt{x}}$

$\int_1^2 e^u \cdot 2du$

$2\int_1^2 e^u du$

4.6 The Natural Logarithmic Function: Integration

Review:

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

$$\int \frac{1}{(x+5)}\,dx$$

$$\frac{d}{dx}[\ln|x|] = \frac{1}{x}$$

$$\frac{d}{dx}[\ln u] = \frac{1}{u}\cdot u' = \frac{u'}{u}$$

THEOREM 4.19 Log Rule for Integration

Let u be a differentiable function of x.

1. $\int \frac{1}{x}\,dx = \ln|x| + C$
2. $\int \frac{1}{u}\,du = \ln|u| + C$

ex: Integrate.

a) $\int \frac{6}{x} dx$

ex: Integrate.

b) $\int \frac{1}{3x+5} dx = \frac{1}{3}\int \frac{1}{u} du = \frac{1}{3}\underline{\ln|3x+5|} + C$

$u = 3x+5$

$du = 3dx$

Check:

$\frac{1}{\cancel{3}} \cdot \frac{1}{3x+5} \cdot \cancel{3}$

$\frac{1}{3x+5}$

ex: Integrate.

c) $\int \frac{2x}{x^2+6}\,dx = \int \frac{1}{u}\,du$

$u = x^2 + 6$
$du = 2x\,dx$

$\ln|x^2+6| + C$

Check

$\frac{1}{x^2+6} \cdot 2x$

ex: Integrate.

d) $\int \frac{\sec^2 x}{\tan x} dx = \int \frac{1}{u} du$

$u = \tan x$

$du = \sec^2 x \, dx$

$\ln|\tan x| + C$

ex: Integrate.

h) $\int \frac{1}{x \ln x} dx = \int \frac{1}{u} du = \ln|u| + c$

$= \ln|\ln x| + c$

$u = \ln x$

$du = \frac{1}{x} dx$

division

$$\int \frac{3x^2-7x+2}{x+5}dx = \int \left(3x-22+\frac{112}{x+5}\right)dx$$

* degree num. $\geq$ degree den.

division

$$\begin{array}{r|rrr} -5 & 3 & -7 & 2 \\ & \downarrow & -15 & 110 \\ \hline & 3 & -22 & 112 \end{array}$$

$$\frac{3x^2}{2} - 22x + 112\ln|x+5| + C$$

ex: Integrate.

1) $\int_0^1 \frac{x-1}{x+1}\,dx$ $= \int_0^1 \left(1 - \frac{2}{x+1}\right)dx$

$$\begin{array}{r|rr} -1 & 1 & -1 \\ & \downarrow & -1 \\ \hline & 1 & -2 \end{array} \qquad \frac{-2}{x+1}$$

$$x - 2\ln|x+1| \Big]_0^1$$

$$(1 - 2\ln 2) - (0 - \cancel{2\ln 1})$$

$\ln 1 = 0$

$1 - 2\ln 2$

$= 1 - \ln 4$

FR 10

The acceleration of a particle moving along a straight line is given by $a = 10e^{2t}$.

(a) Write an expression for the velocity v, in terms of time t, if $v = 5$ when $t = 0$.

(b) During the time that the velocity increases from 5 to 15, how far does the particle travel?

(c) Write an expression for the position s, in terms of time t, of the particle if $s = 0$ when $t = 0$.