2.) $v(2) = \text{initial} + \text{net change}$

$$= \underset{\substack{\uparrow \\ t=1}}{2} + \int_1^2 a(t)\,dt$$

12.)

$f(9) =$ initial + net change

$= 0 + \int_{1}^{9} \frac{(\ln x)^3}{x}\,dx$

t=1

Set up two ways:

Find the average rate of change of f(x) on [1, 3] b.) $v(t)$

$a(4) =$

12d.) Abs. max

t	$x(t)$
0	5
* 1.772	$5 + \int_0^{1.772} v(t)\,dt$
3.06998	$5 + \int_0^{3.0699} v(t)\,dt$
$\sqrt{5\pi}$	$5 + \int_0^{\sqrt{5\pi}} v(t)\,dt$

Check endpoints & rel. max.

HW: 2nd FTC WKST and FR 5, 15, 20

4.4 The FUNdamental Theorem Of Calculus - cont.

ex:

Let h be the function defined by $h(x)=\frac{1}{\sqrt{x^5+1}}$. If g is an antiderivative of h and $g(2)=3$, what is the value of $g(4)$?

(A) -0.020

(B) 0.152

(C) 3.031

(D) 3.152

4.4: FTC Part 2

- Accumulation Functions

The Definite Integral as a Number

Constant

$$\int_a^b f(x)\,dx$$

Constant

f is a function of x.

The Definite Integral as a Function of x

F is a function of x.

$$F(x) = \int_a^x f(t)\,dt$$

Constant

f is a function of t.

In general, an accumulation function comes in the form:

$$F(x) = \int_a^{g(x)} f(t)\,dt,$$

- The 2nd FUNdamental Theorem of Calculus

THEOREM 4.13 The Second Fundamental Theorem of Calculus

If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_a^{g(x)} \underline{f(t)}\,dt\right] = f(g(x)) \cdot g'(x)$$

*The 2nd FUNdamental Theorem of Calculus is used to DIFFERENTIATE an accumulation function.

Find F'(x).

$$F(x) = \int_3^x t^2\, dt$$

$$F(x) = \frac{1}{3}t^3 \Big|_3^x$$

$$F(x) = \frac{1}{3}x^3 - \frac{1}{3}\cdot 3^3$$

$$F'(x) = x^2$$

$$F(x) = \int_7^x t^4\, dt$$

$$F'(x) = x^4$$

Find F'(x)

$$F(x) = \int_3^{x^3} t^2\, dt$$

$$F(x) = \frac{1}{3}t^3 \Big|_3^{x^3}$$

$$(x^2)^3 = x^6$$

$$F(x) = \frac{1}{3}x^9 - \frac{1}{3}\cdot 3^3$$

$$F'(x) = 3x^8 = x^6 \cdot 3x^2$$

Find F'(x)

$$F(x) = \int_{3}^{x^3} t^2 \, dt$$

Find F'(x)

$$F(x) = \int_1^x \sin^3 t \, dt$$

$$F'(x) = \sin^3 x \cdot 1$$

Find F'(x)

$$F(x) = \int_{1}^{x^2} \sqrt{t^2 + 5}\, dt$$

$$F'(x) = \sqrt{x^4 + 5} \cdot 2x$$

Find F'(x)

$$F(x) = \int_{x}^{2} \sqrt{t^2 + 5}\, dt$$

$$F(x) = -\int_{2}^{x} \sqrt{t^2+5}\, dt$$

$$F'(x) = -\sqrt{x^2+5}$$

ex: If $f(x)=\int_1^x \frac{t^4+1}{t}dt$ find $f''(2)$.

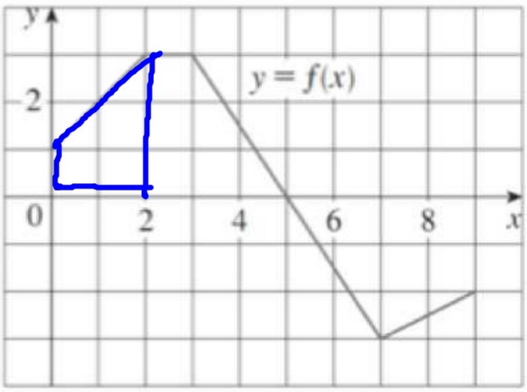
$$f'(x)=\frac{x^4+1}{x}=x^3+x^{-1}$$

$$f''(x)=3x^2-x^{-2}$$

$$f''(2)=12-\frac{1}{4}=\frac{47}{4}=11\,3/4$$

4.4 Notes WKST

The graph of $f(x)$ is shown below. If $g(x)=\int_2^x f(t)\,dt$, evaluate the following or explain why they do not exist.



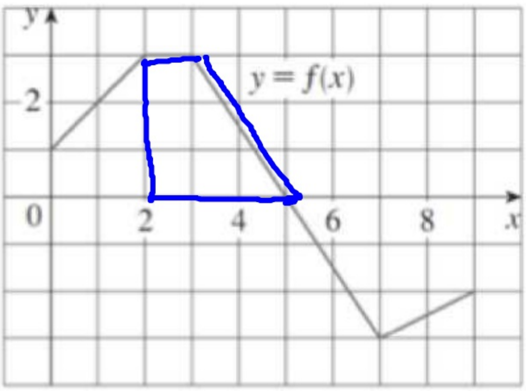
a) $g(0) = \int_2^0 f(t)\,dt$

$= -\int_0^2 f(t)\,dt$

$= -\left(\frac{1}{2}(2)(1+3)\right)$ -4

*See printout.

The graph of $f(x)$ is shown below. If $g(x)=\int_2^x f(t)\,dt$, evaluate the following or explain why they do not exist.



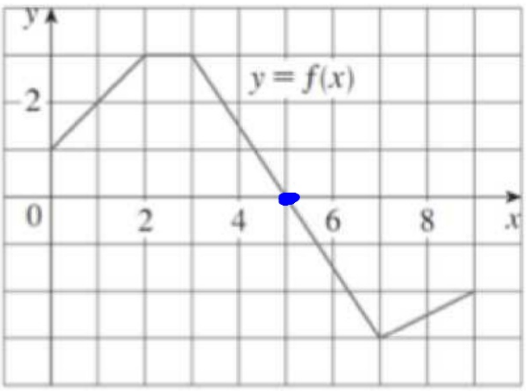
b) $g(5) = \int_2^5 f(t)\,dt$

$= \frac{1}{2}(3)(3+1)$

6

The graph of $f(x)$ is shown below. If $g(x) = \int_2^x f(t)\,dt$, evaluate the following or explain why they do not exist.

c) $g'(5)$



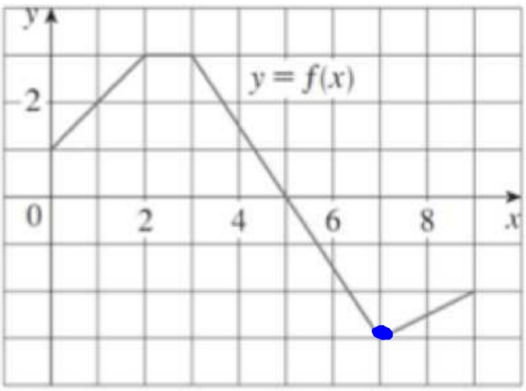
$g'(x) = f(x)$

$g'(5) = f(5)$

$= 0$

The graph of $f(x)$ is shown below. If $g(x)=\int_2^x f(t)\,dt$, evaluate the following or explain why they do not exist.

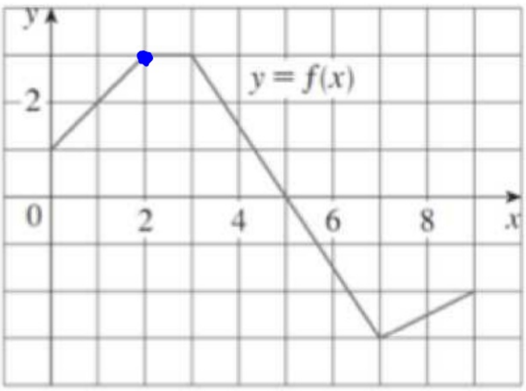
d) $g'(7) = -3$



$* \; g'(x) = f(x)$

$g'(7) = f(7)$

The graph of $f(x)$ is shown below. If $g(x) = \int_2^x f(t)\,dt$, evaluate the following or explain why they do not exist.



e) $g''(2)$

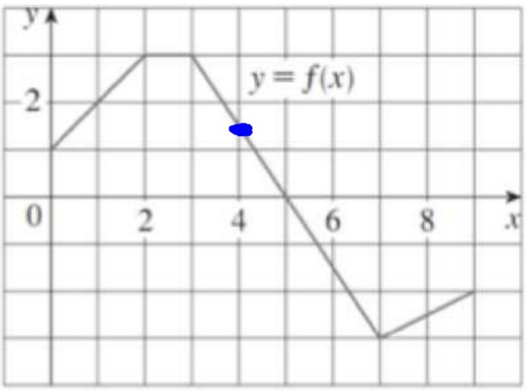
$g'(x) = f(x)$

$g''(x) = f'(x)$

$g''(2) = f'(2)$

dne

The graph of $f(x)$ is shown below. If $g(x) = \int_2^x f(t)\,dt$, evaluate the following or explain why they do not exist.



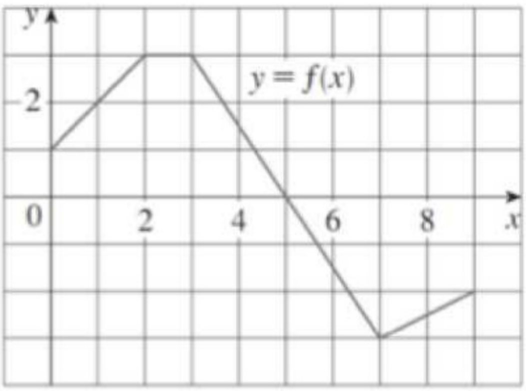
f) $g''(4)$

$g''(x) = f'(x)$

$g''(4) = f'(4)$

$-3/2$

The graph of $f(x)$ is shown below. If $g(x)=\int_2^x f(t)\,dt$, evaluate the following or explain why they do not exist.



g) On what interval does $g(x)$ increase? Justify your answer

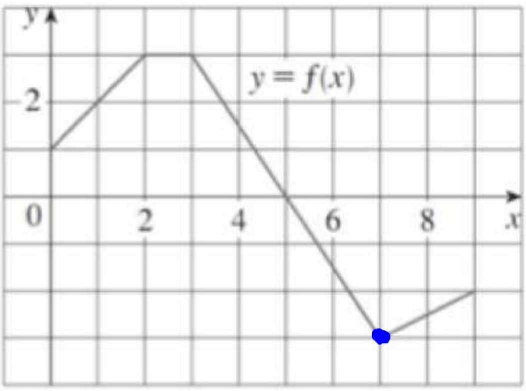
$g'(x) = f(x)$

Since

g is increasing on (0, 5)

because $f(x) > 0$

The graph of $f(x)$ is shown below. If $g(x)=\int_2^x f(t)\,dt$, evaluate the following or explain why they do not exist.



h) At what x-value(s) does $g(x)$ have a point of inflection? Justify your answer.

Since $g'(x) = f(x)$, $g(x)$ has a POI at $x = \underline{7}$ because $\underline{f(x) \text{ changes from}}$ decr. to incr.

ex:

If $f(x)=\int_1^{x^3}\frac{1}{1+\ln t}\,dt$ for $x\geq 1$, then $f'(2)=$

(A) $\frac{1}{1+\ln 2}$

(B) $\frac{12}{1+\ln 2}$

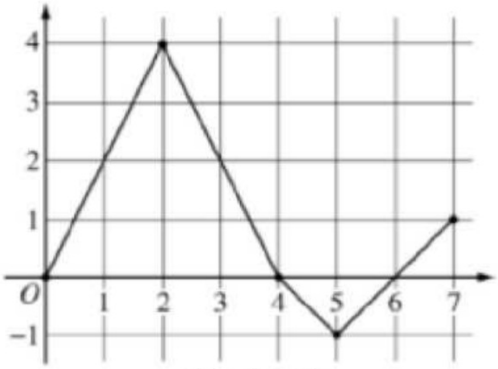
(C) $\frac{1}{1+\ln 8}$

(D) $\frac{12}{1+\ln 8}$

$$f'(x)=\frac{1}{1+\ln x^3}\cdot 3x^2$$

$$f'(2)=\frac{12}{1+\ln 8}$$

FR 16



Graph of f

Let f be a function defined on the closed interval $[0, 7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

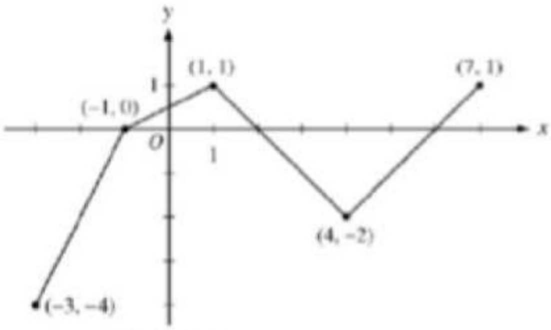
(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b) ? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

FR 13

Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.



Graph of g'

(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

FR 20

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

(c) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicates units of measure.

(d) According to the model f, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?