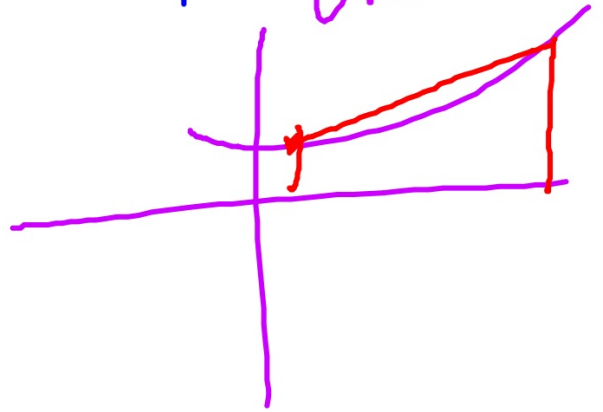


CCD
trap
under ✓



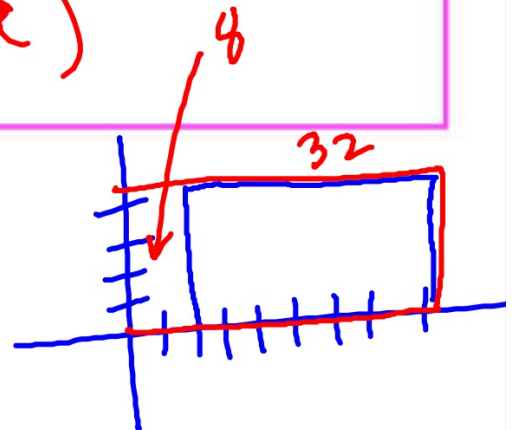
4.4 The FUNdamental Theorem Of Calculus

THEOREM 4.11 The Fundamental Theorem of Calculus

If a function f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on the interval $[a, b]$, then

$$\int_a^b f(x) dx = \underline{F(b) - F(a)}$$

$$\int_2^8 4 dx = 4x + C \Big|_2^8$$
$$(\overset{32}{4 \cdot 8} + \cancel{C}) - (\overset{8}{4 \cdot 2} + \cancel{C})$$
$$24$$



ex: Evaluate.

$$\begin{aligned} \text{a) } \int_0^4 x^2 dx &= \frac{1}{3} x^3 \Big|_0^4 \\ &= \frac{64}{3} - \frac{0}{3} \\ &= \frac{64}{3} \end{aligned}$$

$$\int_0^1 \frac{52x^{7/2} - 66x^{5/2} + 22x^{3/2}}{\sqrt{x}} dx$$

$$\int_0^1 (52x^3 - 66x^2 + 22x) dx$$

$$13x^4 - 22x^3 + 11x^2 + C \Big|_0^1$$

$$13 - 22 + 11$$

$$2$$

ex: Evaluate.

$$\text{b) } \int_1^4 \frac{3}{\sqrt{x}} dx = \int_1^4 3x^{-1/2} dx = \frac{3x^{1/2}}{1/2} = 6x^{1/2} \Big|_1^4$$

(12) - (6)

6

ex: Evaluate.

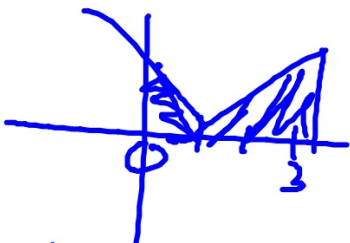
$$c) \int_{\pi/4}^{\pi/3} (\cot^2 x + 1) dx$$

ex: Evaluate.

$$d) \int_0^2 f(x)dx \text{ if } f(x) = \begin{cases} x^4, & x < 1 \\ x^5, & x \geq 1 \end{cases}$$

ex: Evaluate.

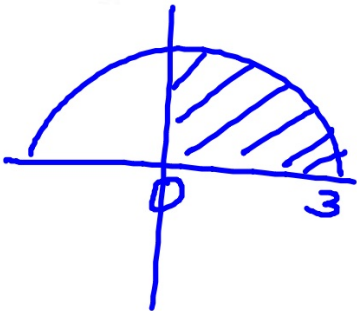
$$e) \int_0^3 |x-1| dx = \int_0^1 (-x+1) dx + \int_1^3 (x-1) dx$$
$$|x-1| = \begin{cases} -x+1, & x \leq 1 \\ x-1, & x > 1 \end{cases}$$



$$\frac{1}{2}(1)(1) + \frac{1}{2}(2)(2)$$
$$\frac{1}{2} + 2$$

ex: Evaluate.

$$f) \int_0^3 \sqrt{9-x^2} dx = \frac{1}{4} \pi \cdot 9$$



- Average Value

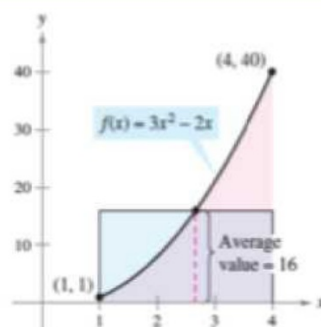
The value $f(c)$ given in the Mean Value Theorem for Integrals is called the average value of f on the interval $[a, b]$.

Definition of the Average Value of a Function on an Interval

If f is integrable on the closed interval $[a, b]$, then the **average value** of f on the interval is

$$\frac{1}{b-a} \int_a^b f(x) dx.$$

See Figure 4.36.



Average Value is not to be confused with Average Rate of Change!!!!

Average Value of $f(x)$ on $[a, b]$:

$$\frac{1}{b-a} \int_a^b f(x) dx$$

~~Average~~ Rate of Change of $f(x)$ on $[a, b]$:

Average

$$\frac{f(b) - f(a)}{b - a}$$

ex: Find the average value of $f(x)$ on the given interval

a) $f(x) = x^2$, $[0, 4]$

$$\frac{1}{4} \int_0^4 x^2 dx$$

$$\frac{1}{4} \cdot \frac{x^3}{3} \Big|_0^4 = \frac{16}{3}$$

ex: Find the average value of $f(x)$ on the given interval

$$\text{b) } f(x) = \begin{cases} x^2 - 3, & x < 3 \\ x + 3, & x \geq 3 \end{cases}, \quad [0, 5]$$

Chocolate-Studded Dream Cookies



*See printout.