4.3 Definite Integration - Geometric Interpretation

- Definite Integral

$$\boxed{\int_a^b f(x)\,dx}$$

where

a - lower limit
b - upper limit

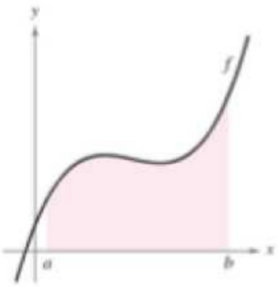
The answer to an indefinite integral is a function

The answer to a definite integral is a numerical value

Definition - If f(x) is <u>continuous</u> and <u>nonnegative</u> on [a,b] then

$$\int_a^b f(x)dx$$

represents the area bounded by f(x), the x-axis, and the lines x=a and x=b.



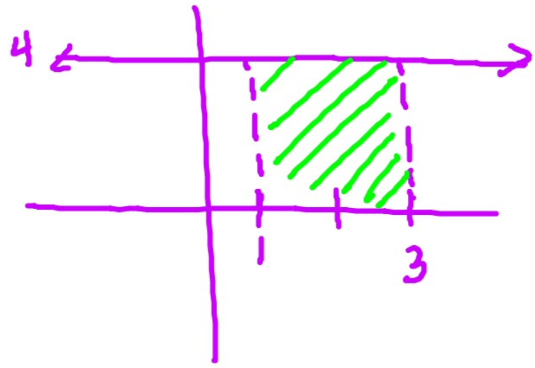
You can use a definite integral to find the area of the region bounded by the graph of f, the x-axis, $x = a$, and $x = b$.

This area is often referred to as "the area under the curve."

ex: Evaluate.

a) $\int_{1}^{3} 4dx = 8$

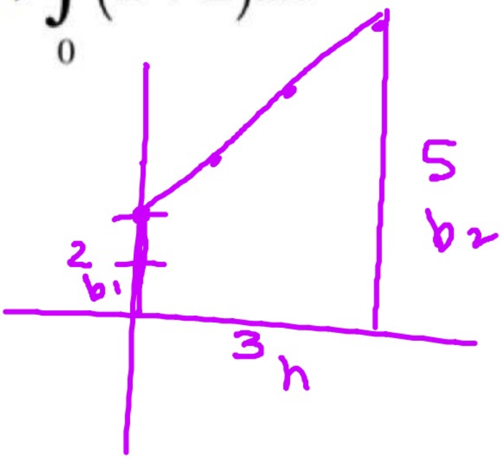
$f(x) = 4$

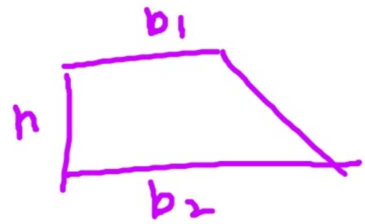


ex: Evaluate.

b) $\int_0^3 (x+2)\,dx = \frac{21}{2}$

$\frac{1}{2}h(b_1+b_2)$





$\frac{1}{2}(3)(2+5)$

$\frac{21}{2}$

ex: Evaluate.

c) $\int_{-2}^{0} \sqrt{4-x^2}\,dx$

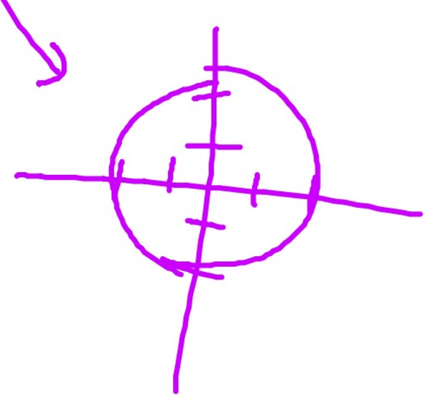
$x^2+y^2=4$

$y=\pm\sqrt{4-x^2}$

$\frac{1}{4}\pi r^2$

$\frac{1}{4}\pi(2)^2$

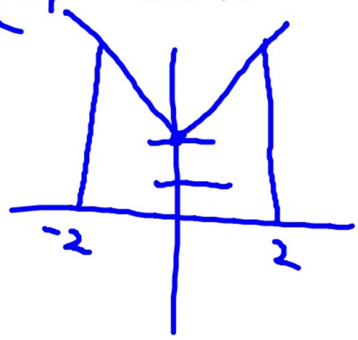
π

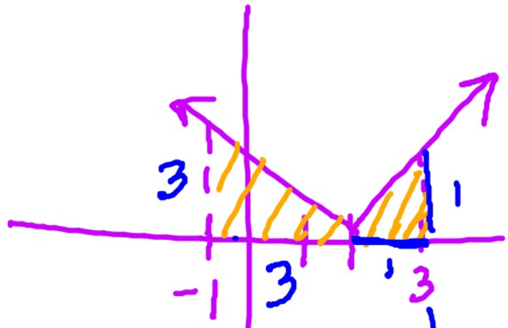


ex: Evaluate.

d) $\int_{-1}^{3} |x-2|\,dx = 5$

$\int_{-2}^{2} (|x|+2)\,dx$



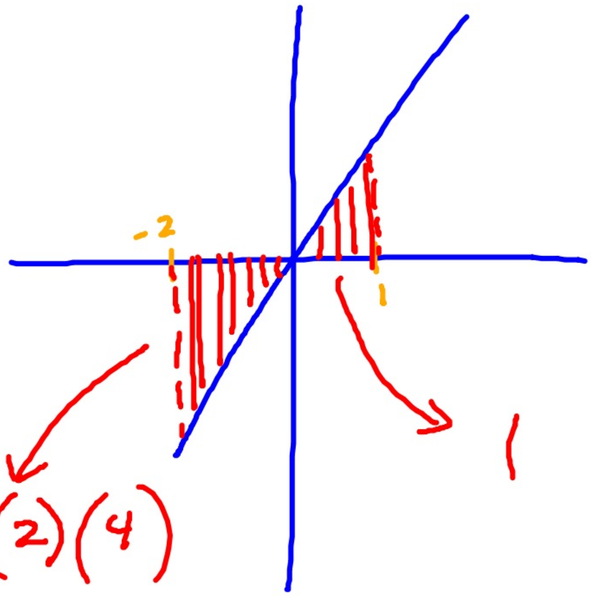


$\frac{1}{2}(3)(3) + \frac{1}{2}(1)(1)$

$\frac{9}{2} + \frac{1}{2}$

ex: Evaluate.

e) $\int_{-2}^{1} 2x\,dx = -3$



$-\frac{1}{2}(2)(4)$

$-4 \quad +1$

-3

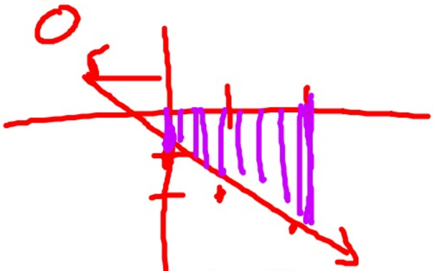
ex: Evaluate.

j) $\int_0^5 f(x)dx$ if $f(x)=\begin{cases} -x-1, & x\le 2 \\ -3, & x>2 \end{cases}$

$\int_0^2 (-x-1)dx + \int_2^5 -3\,dx$

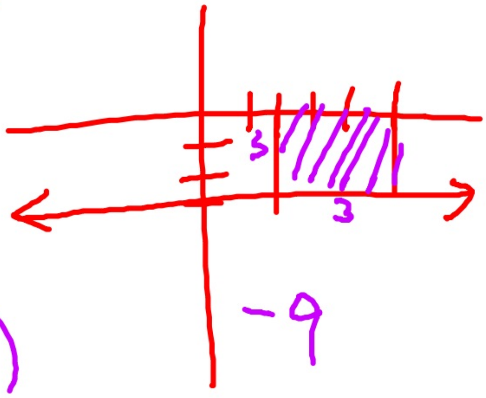
$-4 + -9$

-13



$\frac{1}{2}(2)(1+3)$

-4



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Definite Integral Properties

If f(x) is continuous on [a,b] then...

1. $\int_a^a f(x)dx =$ 0

☆ 2. $\int_b^a f(x)dx =$ $-\int_a^b f(x)dx$

3. $\int_a^b kf(x)dx =$ $K\int_a^b f(x)dx$

Definite Integral Properties - cont.

4. $\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

5. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

$a \le c \le b$

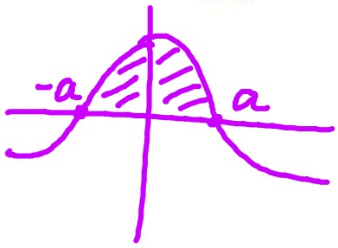
Definite Integral Properties - cont.

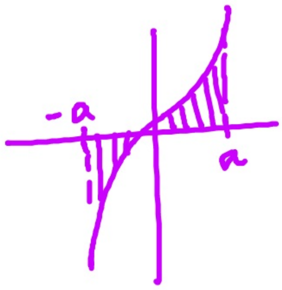
6. $\int_a^b \left(f(x)+c\right)dx = \int_a^b f(x)dx + \int_a^b c\,dx$

7. $\int_{a+c}^{b+c} f(x-c)dx = \int_a^b f(x)dx$

Definite Integral Properties - cont.

8. If f(x) is even, $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$





9. If f(x) is odd, $\int_{-a}^{a} f(x)dx = 0$

ex: If f(x) is continuous and

$$\int_0^1 f(x)dx = -4 \qquad \int_0^3 f(x)dx = 6 \qquad \int_3^5 f(x)dx = -7$$

use the properties of integrals to evaluate.

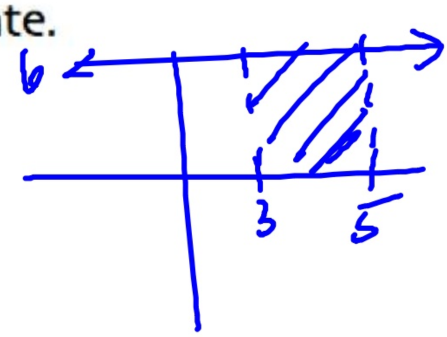
a) $\int_3^0 f(x)dx = -6$

ex: If f(x) is continuous and

$$\int_0^1 f(x)dx = -4 \qquad \int_0^3 f(x)dx = 6 \qquad \int_3^5 f(x)dx = -7$$

use the properties of integrals to evaluate.

b) $\int_3^5 (f(x)+6)dx =$



$$\int_3^5 f(x)dx + \int_3^5 6dx$$

$$-7 + 12 = 5$$

ex: If f(x) is continuous and

$$\int_0^1 f(x)dx = -4 \qquad \int_0^3 f(x)dx = 6 \qquad \int_3^5 f(x)dx = -7$$

use the properties of integrals to evaluate.

c) $\int_0^5 f(x)dx =$ -1

ex: If f(x) is continuous and

$$\int_0^1 f(x)dx = -4 \qquad \int_0^3 f(x)dx = 6 \qquad \int_3^5 f(x)dx = -7$$

use the properties of integrals to evaluate.

d) $\int_5^7 f(x-2)dx =$ -7

ex: If f(x) is continuous and

$$\int_0^1 f(x)dx = -4 \qquad \int_0^3 f(x)dx = 6 \qquad \int_3^5 f(x)dx = -7$$

use the properties of integrals to evaluate.

e) $\int_0^1 3f(x)dx =$ -12

ex: Evaluate.

$$\int_{-13}^{13} \sin x dx = \quad 0$$

ex: If f(x) is even and $\int_{0}^{20} f(x)dx = -7$ then

$$\int_{-20}^{20} f(x)dx =$$

$$2\int_{0}^{20} f(x)dx = -14$$

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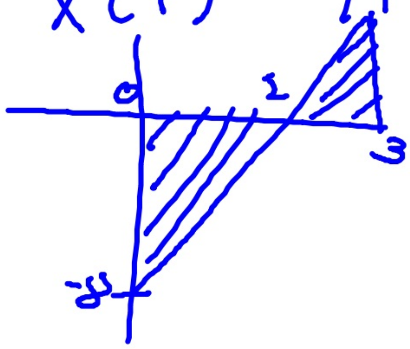
- Total Distance (by calculator)

ex: A particles moves on the x-axis so that its position at any time is given by: $x(t) = \cancel{4t^3 - 18t^2 + 15t - 1}$

$2t^2 - 8t + 3$

Find the total distance traveled by the particle from t=0 to t=3.

$x'(t) = 4t - 8$



$\int_0^2 (4t-8)\,dt = -8$

$\int_2^3 (4t-8)\,dt = 2$

t	$x(t)$
0	3
2	-5
3	-3

8

2

10