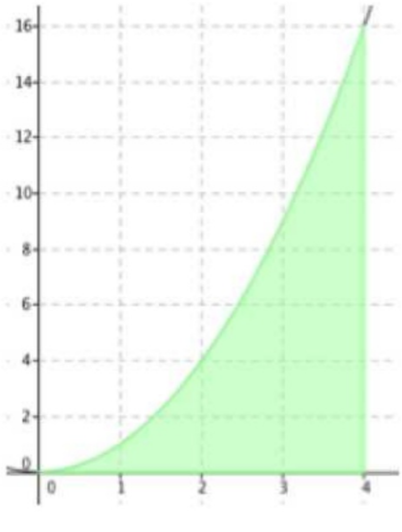
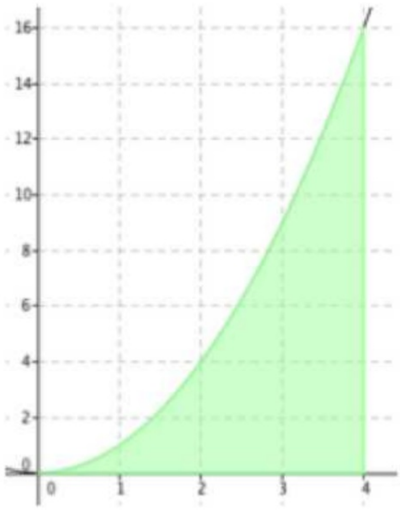
## Riemann Approximations

ex: Evaluate: $\int_0^4 x^2 dx$



Since we can't find the exact area using "shapes", we will approximate the area using a Riemann Approximation.




Approximation Techniques:

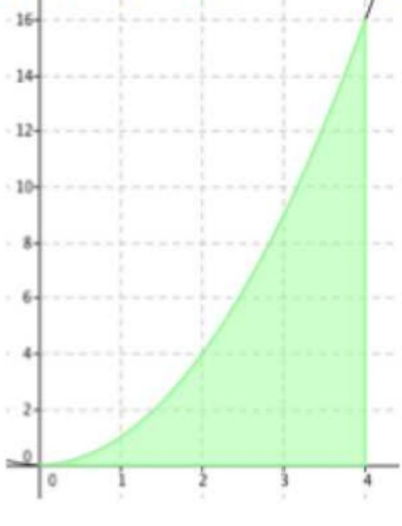
1. Left Riemann
2. Right Riemann
3. Midpoint Riemann
4. Trapezoidal

If there is a constant width, the width can be calculated by:

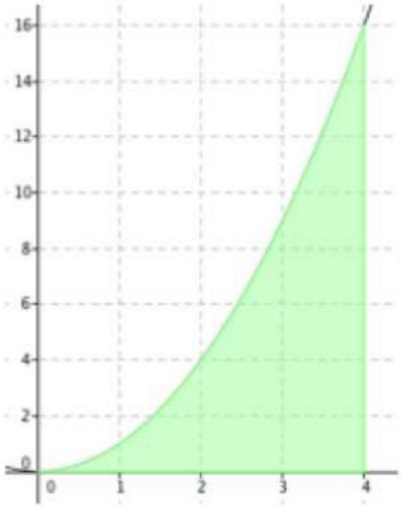
$$Width = (b - a)/n$$

ex: Approximate the integral $\int_0^4 x^2\,dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

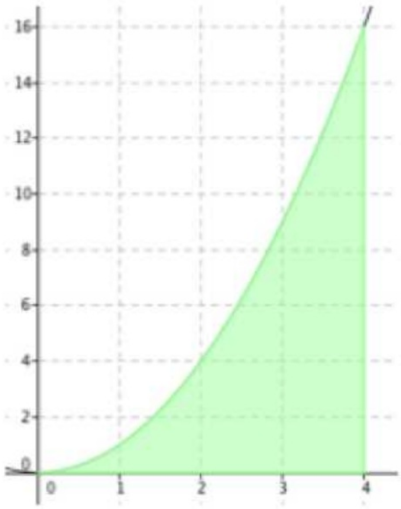
a) Left Riemann, 2 rectangles



b) Right Riemann, 2 rectangles



c) Midpoint Riemann,  2 rectangles



ex: Approximate the integral $\int_{1}^{10} \sqrt{x}dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

a) Left Riemann, 3 rectangles

ex: Approximate the integral $\int_{1}^{10} \sqrt{x}dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

b) Right Riemann, 3 rectangles

ex: Approximate the integral $\int_{1}^{10} \sqrt{x}dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

c) Midpoint Riemann, 3 rectangles

ex: Approximate the integral $\int_{2}^{22} \frac{1}{x} dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

a) Left Riemann, 5 rectangles

ex: Approximate the integral $\int_{2}^{22} \frac{1}{x} dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

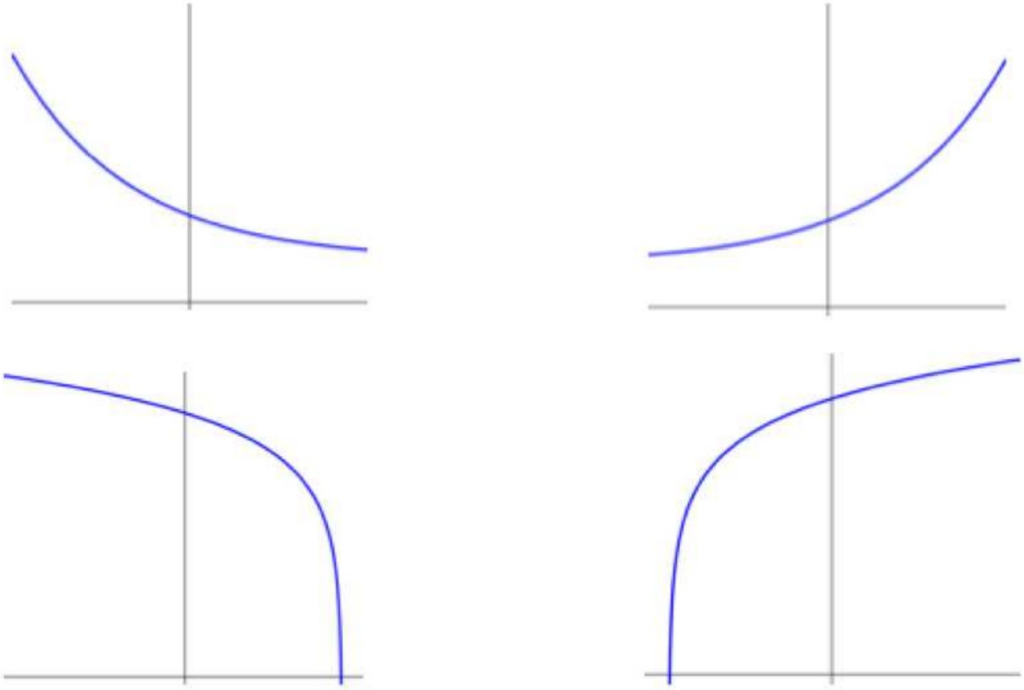
b) Right Riemann, 5 rectangles

ex: Approximate the integral $\int_{2}^{22} \frac{1}{x}dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

c) Midpoint Riemann, 5 rectangles

- Over and Under Estimates

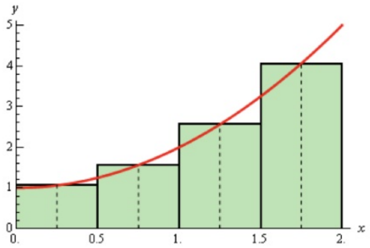
ex: Determine if the Left and Right estimates are over or under approximations.

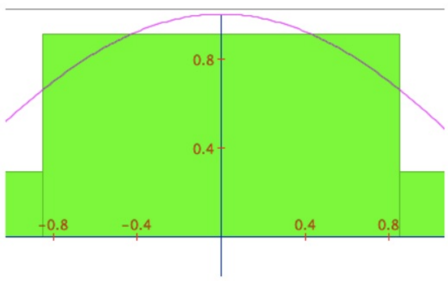


When estimating an integral value using a left or right Riemann Approximation, the approximation will be an over or underestimate depending on whether the curve is

_______________________________________.

For Midpoints:  Will the estimate be an under or over approximation for Concave Up/Concave Down functions




Concave up
Under approx.
more area under
vs. over

Concave down
Over approx.
More area over
vs. under