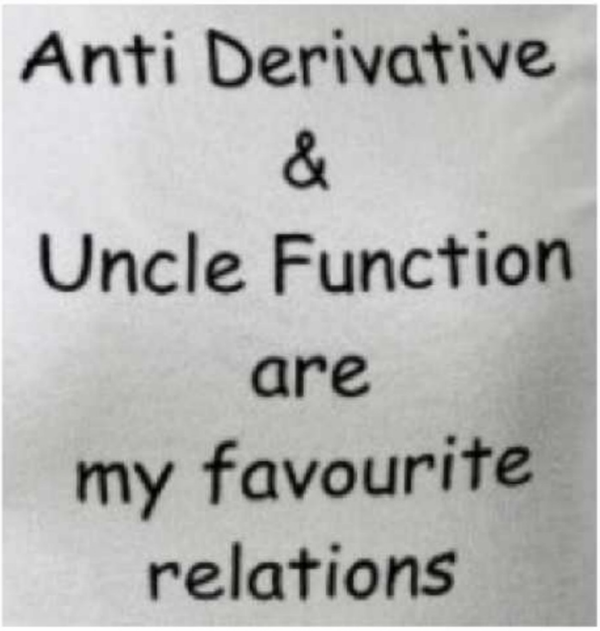
4.1 Antiderivatives and Indefinite Integration



Mock 1: Part 1: Wed. Mar 11 (3:15 - 5:00)
Part 2: Fri. March 13 (during class)

Mock 2: morning of Sat. April 4

119 days until the exam :)

Mock 3: morning of Sat. April 25

Conflicts? (field trips etc) Let me know by the end of this week.

- Antidifferentiation

Definition of Antiderivative

A function F is an **antiderivative** of f on an interval I when $F'(x) = f(x)$ for all x in I.

- Antidifferentiation by Trial and Error!

ex: Find a function, f(x), given its derivative f'(x).

a) $f'(x) = 2x$

$f(x) = x^2$

or

$f(x) = x^2 + 3$

ex: Find a function, f(x), given its derivative f'(x).

b) $f'(x) = 3x^2$

$f(x) = x^3$

c) $f'(x) = x^4$

$f(x) = \frac{1}{5}x^5$

d) $f'(x) = -50\sin x$

$f(x) = 50\cos x$

ex: Find a function, f(x), given its derivative f'(x).

e) $f'(x) = 2\cos 2x$

$f(x) = \sin 2x$

$y = \sin 2x$

$y' = 2\cos 2x$

f) $f'(x) = -x\sin x + \cos x$

Representation of Antiderivatives

If $\frac{d}{dx}[f(x)] = f'(x)$ then $f(x)$ is called the "general antiderivative" of $f'(x)$.

$\frac{3}{2}x^8$

ex: Find the antiderivative: $f'(x) = 12x^7$

$f(x) = 1.5x^8 + C$

- Indefinite Integration

If F(x) is any anti-derivative of f(x) than the most general <u>antiderivative</u> of f(x) is called an <u>indefinite integral</u> and denoted,

$\int x^2 dx$

$$y = \int \underbrace{f(x)}_{\text{Integrand}}\, \overbrace{dx}^{\text{Variable of integration}} = \underbrace{F(x)}_{\text{An antiderivative of } f(x)} + \overbrace{C}^{\text{Constant of integration}}.$$

$\int 3\,dt$

$3t + c$

$\int t^2 dt$

Differentiation and Integration are INVERSE Operations

The inverse nature of integration and differentiation can be verified by substituting $F'(x)$ for $f(x)$ in the indefinite integration definition to obtain

$$\int F'(x)\,dx = F(x) + C.$$

Integration is the "inverse" of differentiation.

Moreover, if $\int f(x)\,dx = F(x) + C$, then

$$\frac{d}{dx}\left[\int f(x)\,dx\right] = f(x).$$

Differentiation is the "inverse" of integration.

These two equations allow you to obtain integration formulas directly from differentiation formulas, as shown in the following summary.

- Basic Rules

Differentiation Rules	Integration Rules
$\frac{d}{dx}[kx]=$ K	$\int k\,dx=$ $Kx+C$
$\frac{d}{dx}[kf(x)]=$ $Kf'(x)$	$\int kf(x)dx=$ $KF(x)+C$
$\frac{d}{dx}[f(x)\pm g(x)]=$ $f'(x)\pm g'(x)$	$\int[f(x)\pm g(x)]dx=$ $F(x)\pm G(x)+C$
$\frac{d}{dx}[x^n]=$ nx^{n-1}	$\int x^n\,dx=\frac{1}{n+1}x^{n+1}+C$, $(n\neq -1)$

$\frac{x^{n+1}}{n+1}+C$ $\int x^2dx=\frac{1}{3}x^3+C$

*See printout.

Differentiation Rules	Integration Rules
$\frac{d}{dx}[\sin x] =$	$\int \sin x \, dx =$ $-\cos x + C$
$\frac{d}{dx}[\cos x] =$	$\int \cos x \, dx =$ $\sin x + C$
$\frac{d}{dx}[\tan x] =$	$\int \sec^2 x \, dx =$ $\tan x + C$
$\frac{d}{dx}[\csc x] =$	$\int \csc x \cot x \, dx =$ $-\csc x + C$
$\frac{d}{dx}[\sec x] =$	$\int \sec x \tan x \, dx =$ $\sec x + C$
$\frac{d}{dx}[\cot x] =$	$\int \csc^2 x \, dx =$ $-\cot x + C$

Differentiation Rules	Integration Rules
$\frac{d}{dx}[e^x] =$ e^x	$\int e^x\,dx =$ $e^x + C$
$\frac{d}{dx}[a^x] =$ $\ln a \cdot a^x$	$\int a^x\,dx =$ $\frac{1}{\ln a} \cdot a^x + C$
$\frac{d}{dx}[\ln x] =$ $\frac{1}{x}$	$\int \frac{1}{x}\,dx =$ $\ln\lvert x\rvert + C$

ex: Evaluate.

a) $\int x^5\,dx =$ $\frac{1}{6}x^6 + C$ or $\frac{x^6}{6} + C$

b) $\int 5\sec^2 x\,dx =$ $5\int \sec^2 x\,dx = 5\tan x + C$

c) $\int \sqrt{x}\,dx =$ $\int x^{1/2}\,dx = \frac{2}{3}x^{3/2} + C$

ex: Evaluate.

d) $\int \frac{8}{x^2} dx =$ $8\int x^{-2} dx = \frac{8x^{-1}}{-1} + C$

$= -\frac{8}{x} + C$

e) $\int \frac{8}{x} dx =$ $8\int \frac{1}{x} dx = 8\ln|x| + C$

f) $\int 2x\, dx =$

ex: Evaluate.

g) $\int \frac{1}{(2x)^3} dx =$ $\frac{1}{8}\int x^{-3} dx = \frac{1}{8}\frac{x^{-2}}{-2} + C$

$-\frac{1}{16}x^{-2} + C$

h) $\int \left(4x^3 - 3^x + \sin x - 5e^x\right) dx =$ $x^4 - \frac{3^x}{\ln 3} - \cos x - 5e^x + C$

i) $\int \frac{x^7 - 5x^3 + 2x}{x^4} dx =$ $\frac{1}{4}x^4 - 5\ln|x| - x^{-2} + C$

$\int \left(x^3 - \frac{5}{x} + 2x^{-3}\right) dx$

ex: Evaluate.

j) $\int (1+3x)x^2\,dx =$

$\int (x^2+3x^3)dx = \frac{1}{3}x^3 + \frac{3}{4}x^4 + C$

k) $\int \frac{\sin x}{\cos^2 x}dx = \int \frac{\sin x}{\cos x}\frac{1}{\cos x}dx = \int \tan x \sec x\,dx$

$= \sec x + C$

l) $\int \left(1+\cot^2 x\right)dx = \int \csc^2 x\,dx = -\cot x + C$

- Differential Equations

  A differential equation is an equation involving a derivative.

- Differential Equations Have 2 Types of Solutions

  1. General Solution - general antiderivative

     $+C$

  2. Particular Solution - an antiderivative that passes through a given initial condition.

ex: $f'(x) = 3x^2 - 1$

a) Find the general solution.

$f(x) = x^3 - x + C$

b) Find the particular solution that satisfies the intitial condition $f(2) = 4$.

$4 = 2^3 - 2 + C$

$-2 = C$

$f(x) = x^3 - x - 2$

- Total Distance (by hand)

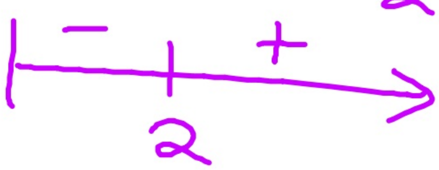
ex: A particles moves on the x-axis so that its position at any time is given by: $x(t) = \cancel{4t^3 - 18t^2 + 15t - 1}$

$2t^2 - 8t + 3$

Find the total distance traveled by the particle from t=o to t=3.

$x'(t) = 4t - 8$

$2 = t$



t	$x(t)$
0	3
2	−5
3	−3

8

2

10

FR 2

A particle moves on the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 12t^2 - 36t + 15$. At $t = 1$, the particle is at the origin.

(a) Find the position $x(t)$ of the particle at any time $t \geq 0$.

(b) Find all values of t for which the particle is at rest.

(c) Find the maximum velocity of the particle for $0 \leq t \leq 2$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

4.1-4.3 Extra Practice WKST

1.

If $f'(x) = 12x^2 - 6x + 1$, $f(1) = 5$, then $f(0)$ equals

(A) 2 (B) 3 (C) 4 (D) -1 (E) 0

$f(x) = 4x^3 - 3x^2 + x + C$

$5 = 4 - 3 + 1 + C$

$3 = C$

$f(x) = 4x^3 - 3x^2 + x + 3$

*See printout.

4.1-4.3 Extra Practice WKST

2.

Find all functions g such that $g'(x) = \dfrac{5x^2 + 4x + 5}{\sqrt{x}}$

(A) $g(x) = 2\sqrt{x}\left(x^2 + \frac{4}{3}x - 5\right) + C$ (B) $g(x) = 2\sqrt{x}\left(x^2 + \frac{4}{3}x + 5\right) + C$

(C) $g(x) = 2\sqrt{x}\left(5x^2 + 4x - 5\right) + C$ (D) $g(x) = \sqrt{x}\left(x^2 + \frac{4}{3}x + 5\right) + C$

(E) $g(x) = \sqrt{x}\left(5x^2 + 4x + 5\right) + C$

4.1-4.3 Extra Practice WKST

3.

$f''(t) = 6t + 2$

Determine $f(t)$ when $f''(t) = 2(3t+1)$ and $f'(1) = 3$, $f(1) = 5$.

(A) $f(t) = 3t^3 - 2t^2 + 2t + 2$ (B) $f(t) = t^3 - 2t^2 + 2t + 4$

(C) $f(t) = 3t^3 + t^2 - 2t + 3$ (D) $f(t) = t^3 - t^2 + 2t + 3$

(E) $f(t) = t^3 + t^2 - 2t + 5$

$f'(t) = 3t^2 + 2t + C$

$3 = 3 + 2 + C$

$-2 = C$

$f'(t) = 3t^2 + 2t - 2$

$f(t) = t^3 + t^2 - 2t + C$

$5 = C$

4.1-4.3 Extra Practice WKST

4.

$\boxed{\sin 2x = 2\sin x\cos x}$

$f(x) = \sin x\cos x$

Consider the following functions:

I. $F_1(x) = \dfrac{\sin^2 x}{2}$

II. $F_2(x) = -\dfrac{\cos 2x}{4}$

III. $F_3(x) = -\dfrac{\cos^2 x}{2}$

Which are antiderivatives of $f(x) = \sin x \cos x$? (Hint: take the derivative of each and manipulate)

(A) II only (B) I only (C) I & III only (D) I, II, & III (E) I & II only

$\frac{2}{2}(\sin x)(\cos x)$

$\frac{1}{2}\sin 2x$

$+\cos x\sin x$

4.1-4.3 Extra Practice WKST

5.

A particle moves along the x-axis. The velocity of the particle at time t is $6t - t^2$. What is the total distance traveled by the particle from time $t = 0$ to $t = 3$?

(A) 3 (B) 6 (C) 9 (D) 18 (E) 27

4.1-4.3 Extra Practice WKST

6.

A particle moves along the x-axis so that its acceleration at time t is $a(t) = 8 - 8t$ in units of feet and seconds. If the velocity of the particle at $t = 0$ is 12 ft/sec, how many seconds will it take for the particle to reach its furthest point to the right?

(A) 6 seconds (B) 5 seconds (C) 3 seconds (D) 7 seconds (E) 4 seconds