

$$39.) f(x) = x^4 - 4x^3 + 2$$

$$f'(x) = 4x^3 - 12x^2$$

$$0 = 4x^2(x-3)$$

$$x = 0, 3$$

$$f''(x) = 12x^2 - 24x$$
$$= 12x(x-2)$$

$$f''(0) = 0; \text{ test fails}$$

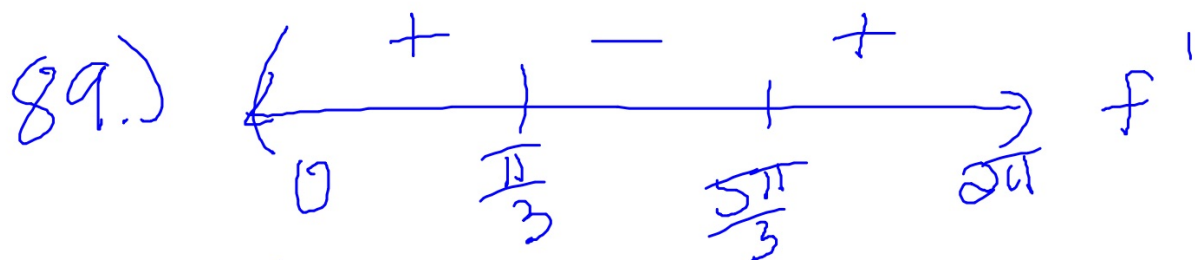
~~$$(0, 2)$$~~

$$f''(3) > 0$$

rel. min

$$(3, -25)$$





$$f(x) = 2 \sin x - x$$

$$f'(x) = 2 \cos x - 1$$

$$f''(x) = -2 \sin x$$

$$0 = -2 \sin x$$

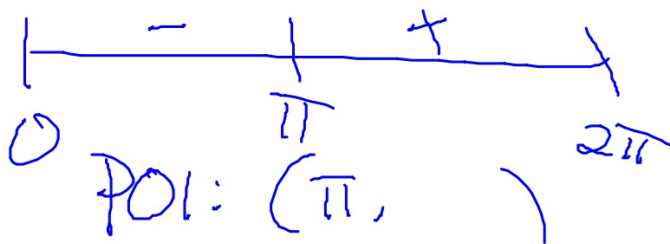
$$0, \pi, 2\pi = x$$

$$\left(\frac{\pi}{3}, \sqrt{3} - \frac{\pi}{3}\right)$$

rel. max.

rel. min

$$\left(\frac{5\pi}{3}, -\sqrt{3} - \frac{5\pi}{3}\right)$$



3.5 Summary of Curve Sketching

To sketch curves you must find:

- x & y intercepts
- relative extrema
- intervals of increasing/decreasing
- concavity
- points of inflections
- asymptotes
- end behavior
- symmetry

For Rational Functions, also find:

- holes

$$y = \frac{x-1}{x^2-1} = \frac{1}{x+1}$$

hole @ $x=1$

For Radical Functions, also find:

- domain

$$y = \sqrt{9-x^2}$$

For Polynomial Functions, also find:

- end behavior

\uparrow \downarrow
 $-x^3$

\downarrow \uparrow
 x^3

\uparrow \uparrow
 x^2

\downarrow \downarrow
 $-x^2$

$$y = x^6 - x^7 + 3$$

\uparrow \downarrow

Sketching Polynomial Functions

ex: Sketch.

$$D: (-\infty, \infty)$$

EB: $\uparrow\uparrow$

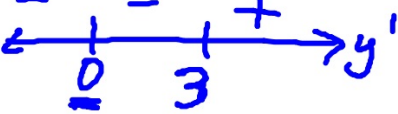
$$x\text{-int: } (0, 0) (4, 0)$$

$$y\text{-int: } (0, 0)$$

$$y' = 4x^3 - 12x^2$$

$$0 = 4x^2(x-3)$$

rel. min $(3, -27)$



$$y'' = 12x^2 - 24x$$

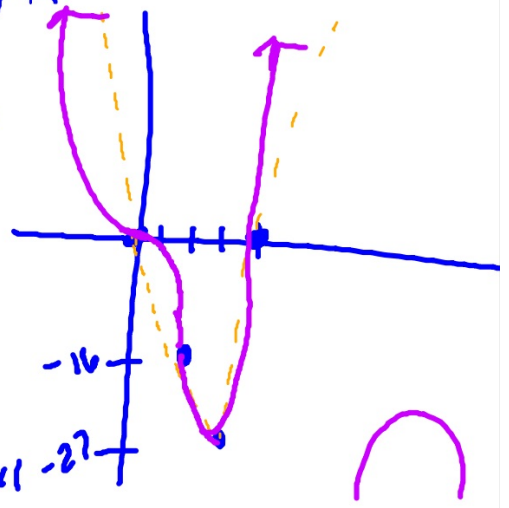
$$= 12x(x-2)$$



PVI: $(0, 0)$
 $(2, -16)$

$$y = x^4 - 4x^3 = x^3(x-4)$$

$$y = x^5 - 5x$$



Sketching Rational Functions

ex: Sketch.

VA: $x = 2$
no x-int.
y-int $(0, -2)$
EB:
 $\lim_{x \rightarrow \infty} f(x) = \infty$
 $\lim_{x \rightarrow -\infty} f(x) = -\infty$

$$y = \frac{x^2 - 2x + 4}{x - 2}$$

$$y' = \frac{x(x-4)}{(x-2)^2}$$

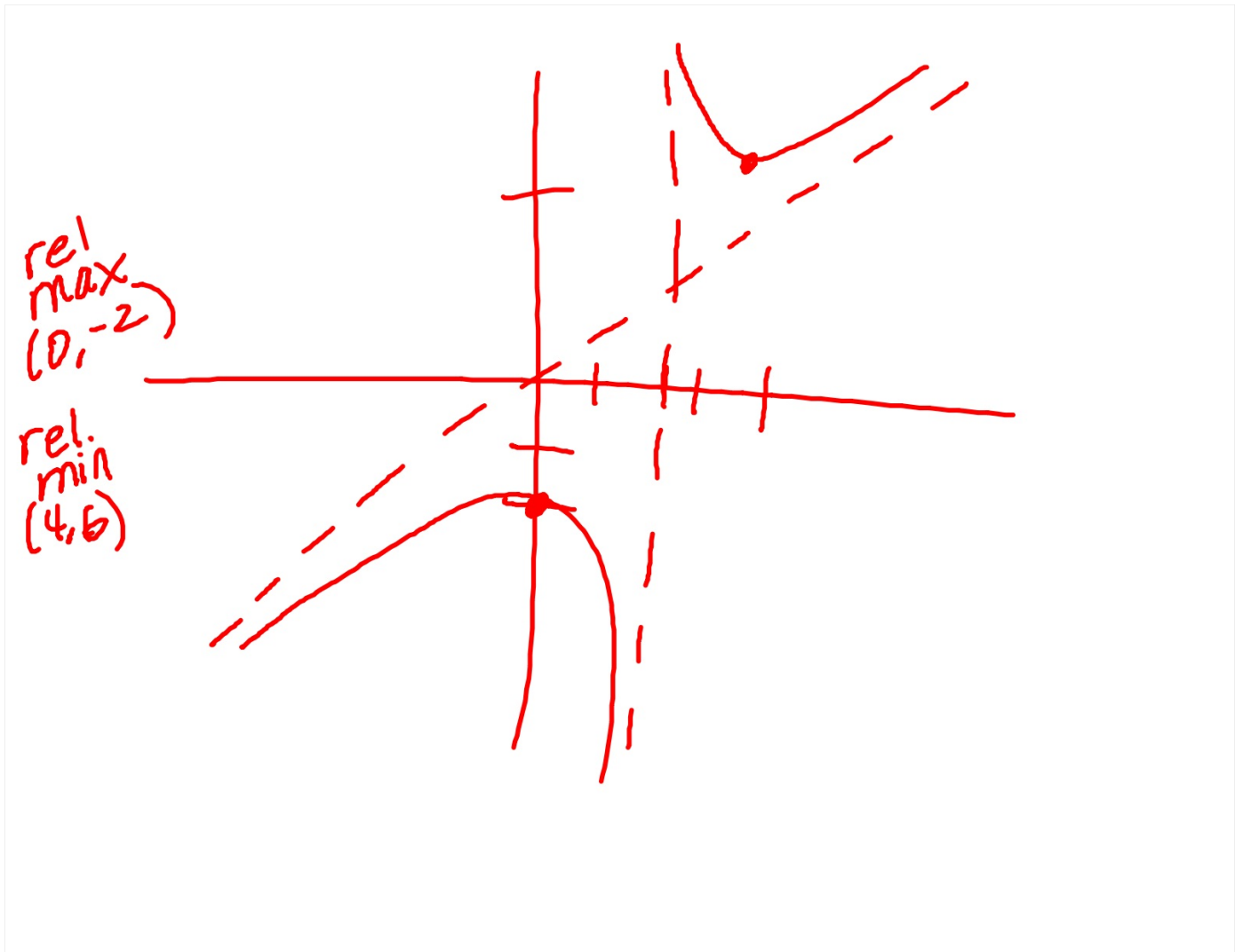
$$y'' = \frac{8}{(x-2)^3}$$

FRQ
4, 13

SA?

2	1	-2	4
		2	0
	1	0	X

SA: $y = x$



$$f(-x) = f(x) \text{ even}$$

$$f(x) = x^4 + x^2$$

$$f(-x) = (-x)^4 + (-x)^2$$

$$f(-x) = -f(x) \text{ odd}$$

$$f(x) = x^3 - x$$

$$f(-x) = -x^3 + x$$

$$= -(x^3 - x)$$

Sketching Radical Functions

ex: Sketch.

$$y = x\sqrt{9 - x^2}$$

Sketching Radical Functions

$D: (-\infty, \infty)$

ex: Sketch.

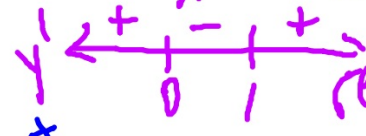
$x_{int}: x^{2/3} (2x - 3)$
 $x = 0 \quad x = \frac{27}{8}$
 ~~$y = 2x^{5/3} - 5x^{4/3}$~~

$2x - 3x^{2/3}$

$y_{int} (0, 0)$

$y' = 2 - 2x^{-1/3} = \frac{2x^{1/3} - 2}{x^{1/3}}$

$y'' = \frac{2}{3} x^{-4/3}$



rel max (0, 0)
rel min (1, -1)



ex: Sketch.

$$y = (\arctan x)^2$$

ex: Sketch.

$$y = \ln(x^2 + 2x + 3)$$

ex: Sketch.

$$y = \frac{1}{1 + e^{-x}}$$