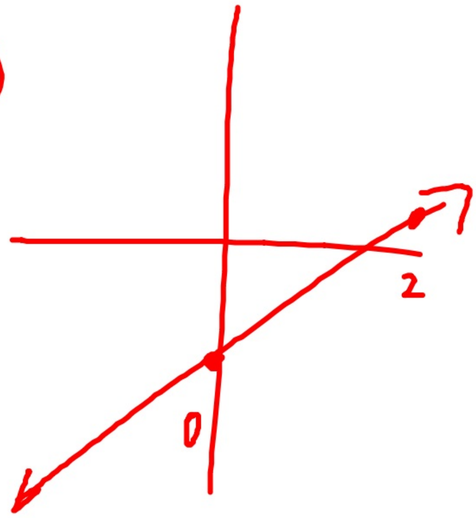
30.) $h(t) = \frac{t}{t+3}$

$[-1, 6]$

$h'(t) = \frac{3}{(t+3)^2}$

t	$h(t)$	
-1	$-1/2$	min
6	$2/3$	max

47.)



[0,2] ✓

[0,2)

(0,2]

(0,2)

41.) $h(x) = 5e^{x} - e^{2x}$

$h'(x) = 5e^{x} - 2e^{2x}$

$0 = e^{x}(5 - 2e^{x})$

$5 = 2e^{x}$

$\frac{5}{2} = e^{x}$

x	y	
-1	$\frac{5}{e} - \frac{1}{e^2}$	
$\ln\frac{5}{2}$	$25/4$	☆ Max
2	$5e^2 - e^4$	Min

43.) $y = e^x \sin x$

$y' = e^x \cos x + \sin x e^x$

$y' = e^x(\cos x + \sin x)$

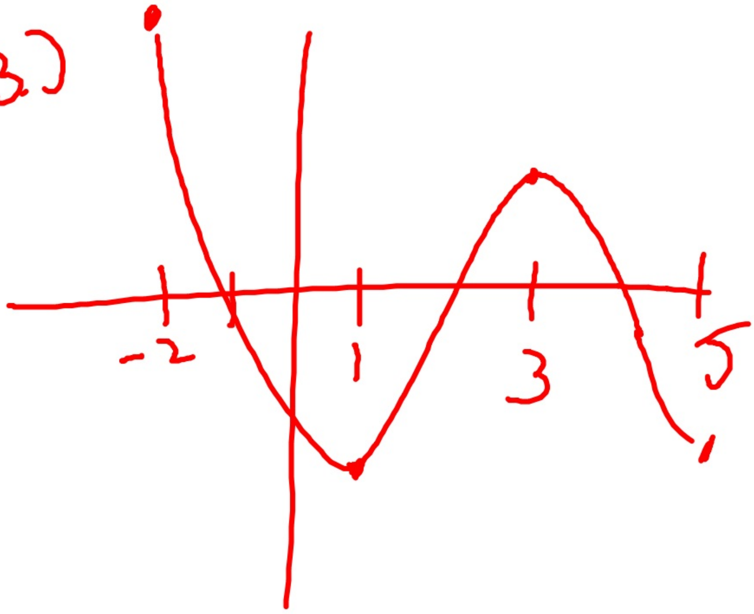
$\cos x + \sin x = 0$

$x = \frac{3\pi}{4}$ → $\cos x = -\sin x$

$-1 = \tan x$

x	y
0	0
$3\pi/4$	$e^{3\pi/4}\frac{\sqrt{2}}{2}$
π	0

(63)



3.2 Rolle's Theorem and the Mean Value Theorem

ex: List the critical numbers of f(x).

$$f(x) = x^{4/5}(x-5)^2$$

ex: Find the maximum and miniumum values of f(x) on the indicated interval.

$$f(x) = \sin x + \cos x, \quad \left[0, \frac{\pi}{2}\right]$$

THEOREM 3.4 The Mean Value Theorem

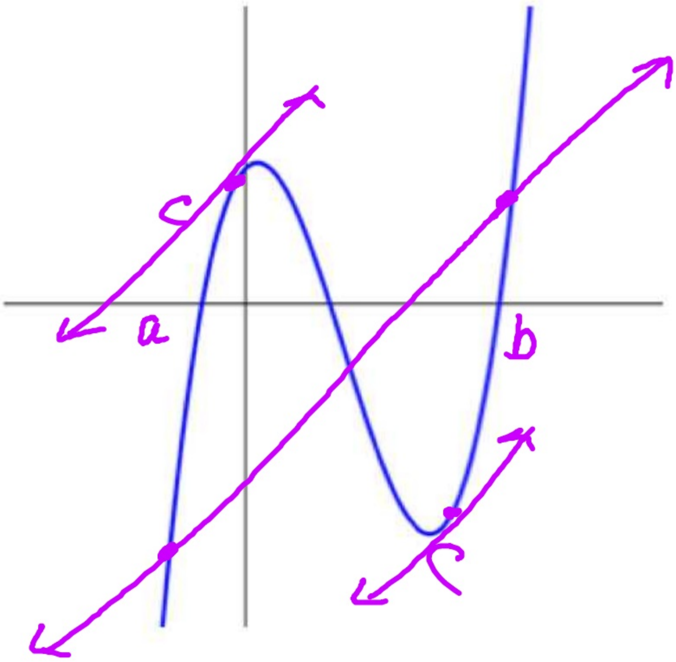
If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$m_{tan} = m_{sec}$$

The Graphical Interpretation of the Mean Value Theorem.

(MVT)



ex: Determine the value(s) of c guaranteed by the conclusion of the MVT on the given interval, if possible.

a) $f(x) = 5 - \dfrac{4}{x}$, $[1,4]$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Cont ✓ $-4x^{-1}$

diff ✓

$$f'(x) = \frac{4}{x^2}$$

$$\frac{4}{c^2} = \frac{4 - 1}{4 - 1}$$

$$\frac{4}{c^2} = 1 \qquad c^2 = 4$$

$$c = \pm 2$$

$$c = 2$$

ex: Determine the value(s) of c guaranteed by the conclusion of the MVT on the given interval, if possible.

b) $f(x) = \sqrt{x-4}, \quad [4,8]$

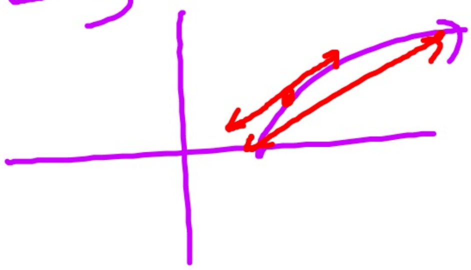
cont ✓

diff ✓

$f'(x) = \frac{1}{2\sqrt{x-4}}$

$\frac{1}{2\sqrt{c-4}} = \frac{1}{2}$

$c = 5$



ex: Determine the value(s) of c guaranteed by the conclusion of the MVT on the given interval, if possible.

c) $f(x) = \dfrac{1}{x}, \quad [-1, 1]$

f(x) is not cont.
at x=0

MVT does not apply

ex: Determine the value(s) of c guaranteed by the conclusion of the MVT on the given interval, if possible.

d) $f(x)=|x|, \quad [-2,2]$

cont ✓

diff $(-2;2)$

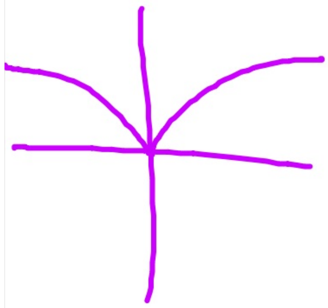
No

MVT does not apply

e.) $f(x) = x^{2/3}$ $[0, 8]$

cont ✓

diff $(0, 8)$ ✓



$$\frac{2}{3x^{1/3}} = \frac{1}{2}$$

$$3x^{1/3} = 4$$

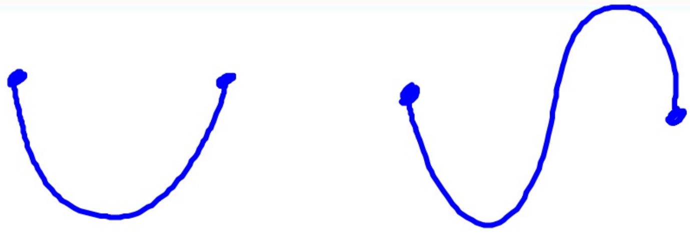
$$x^{1/3} = \frac{4}{3}$$

$$x = \frac{64}{27}$$

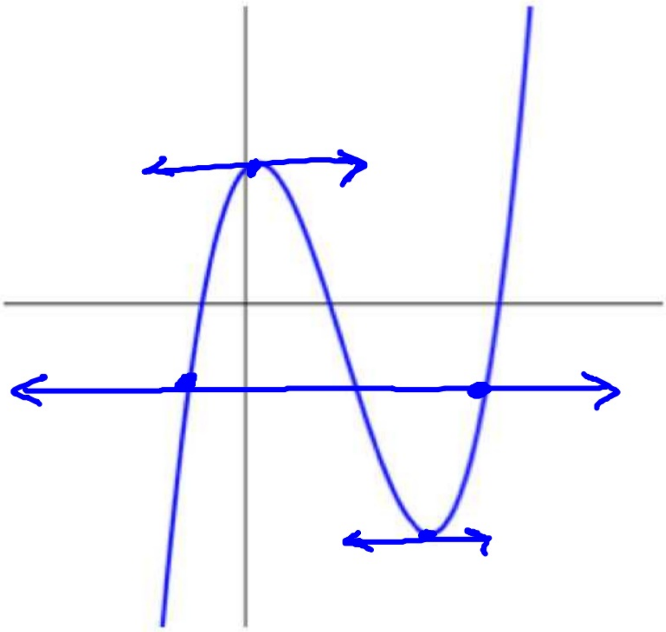
THEOREM 3.3 Rolle's Theorem

Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If $f(a) = f(b)$, then there is at least one number c in (a, b) such that

$$f'(c) = 0$$



The Graphical Interpretation of Rolle's Theorem.



ex: Determine the value(s) of c guaranteed by the conclusion of Rolle's Theorem on the given interval, if possible.

a) $f(x) = \cos\frac{x}{3}, \quad [0, 6\pi]$

$f'(x) = -\frac{1}{3}\sin\frac{x}{3}$

Cont. ✓
diff ✓
$f(0) = 1$ ✓
$f(6\pi) = 1$

$-\frac{1}{3}\sin\frac{x}{3} = 0$

$\sin\frac{x}{3} = 0$

$\frac{x}{3} = 0, \pi, 2\pi, 3\pi, 4\pi$

$x = 0, 3\pi, 6\pi, \cancel{9\pi}, \ldots$

$C = 3\pi$

ex: Determine the value(s) of c guaranteed by the conclusion of Rolle's Theorem on the given interval, if possible.

b) $f(x) = x, \quad [1, 20]$

$f(1) \neq f(20)$

Rolle's Thm
does not apply

ex:

Let f be the function given by $f(x) = x^3 - 3x^2$. What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0,3]$?

(A) 0 only (B) 2 only (C) 3 only (D) 0 and 3 (E) 2 and 3

$$3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$$x = \cancel{0}, 2$$

$$\frac{f(3)-f(0)}{3-0} = \frac{0}{3}$$

FR:

Let f be the function given by $f(x) = x^3 - 7x + 6$. $f(x) = x^3 - 7x + 6$

(a) Find the zeros of f.

(b) Write an equation of the line tangent to the graph of f at $x = -1$.

(c) Find the number c that satisfies the conclusion of the Mean Value Theorem for f on the closed interval $[1, 3]$.

$$1 \,|\, 1 \quad 0 \quad -7 \quad 6$$

$$c = \sqrt{\frac{13}{3}}$$