30.) $h(t) = \dfrac{t}{t+3}$ $[-1, 6]$

$h'(t) = \dfrac{3}{(t+3)^2}$

t	$h(t)$	
-1	$-1/2$	min
6	$2/3$	max

39.) $f(x) = \arctan x^2$

$$f'(x) = \frac{2x}{1+x^4}$$

x	$f(x)$	
-2	$\arctan 4$	max
0	0	min
1	$\pi/4$	

17.) $f(t) = te^{-2t}$

$f'(t) = t(-2e^{-2t}) + e^{-2t} \cdot 1$

$0 = -e^{-2t}(2t-1)$

$\downarrow \qquad \downarrow$

$\emptyset \qquad t = \frac{1}{2}$

76.) $f(x) = \frac{4\ln x}{x^3}$ $x > 0$

$f'(x) = \frac{x^3 \cdot \frac{4}{x} - 4\ln x \cdot 3x^2}{x^6}$

$= \frac{4x^2 - 4\ln x \cdot 3x^2}{x^6}$

$= \frac{4x^2(1 - 3\ln x)}{x^6}$

$1 = 3\ln x$

$\frac{1}{3} = \ln x$

$e^{\frac{1}{3}} = e^{\ln x}$

$x = e^{1/3}$

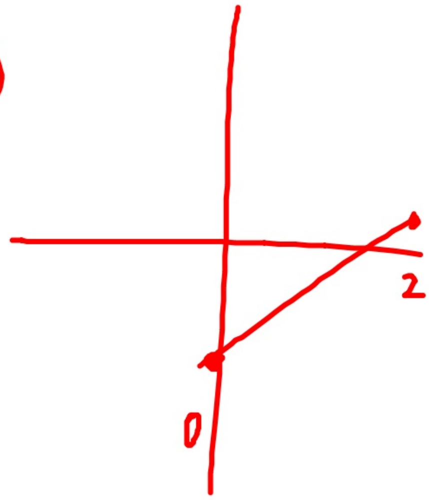
75.) $f(x) = x^2(3x-1)^3$

$$f'(x) = \underbrace{x^2 \cdot 3(3x-1)^2 \cdot 3} + \underbrace{(3x-1)^3 \cdot 2x}$$

$$0 = x(3x-1)^2\left[9x + 2(3x-1)\right]$$

$$0 = x(3x-1)^2\left[15x-2\right]$$

47.)



[0,2] ✓

[0,2)

(0,2]

(0,2)

41.) $h(x) = 5e^{x} - e^{2x}$

$h'(x) = 5e^{x} - 2e^{2x}$

$0 = e^{x}(5 - 2e^{x})$

$5 = 2e^{x}$

$\frac{5}{2} = e^{x}$

$\ln\frac{5}{2} = x$

x	y	
-1	$\frac{5}{e} - \frac{1}{e^2}$	
$\ln\frac{5}{2}$	$25/4$	☆ Max
2	$5e^2 - e^4$	Min

$2\sin x\cos x = \sin 2x$

77.) $g(x) = \sin x\cos x = \frac{1}{2}(2\sin x\cos x) = \frac{1}{2}\sin 2x$

$g'(x) = \sin x(-\sin x) + \cos x \cdot \cos x$

$0 = \cos^2 x - \sin^2 x$

$\sin^2 x = \cos^2 x$

$\sqrt{\tan^2 x} = \sqrt{1}$

$\tan x = \pm 1$

$y' = \cos 2x$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

43.) $y = e^x \sin x$

$y' = e^x \cos x + \sin x e^x$

$y' = e^x(\cos x + \sin x)$

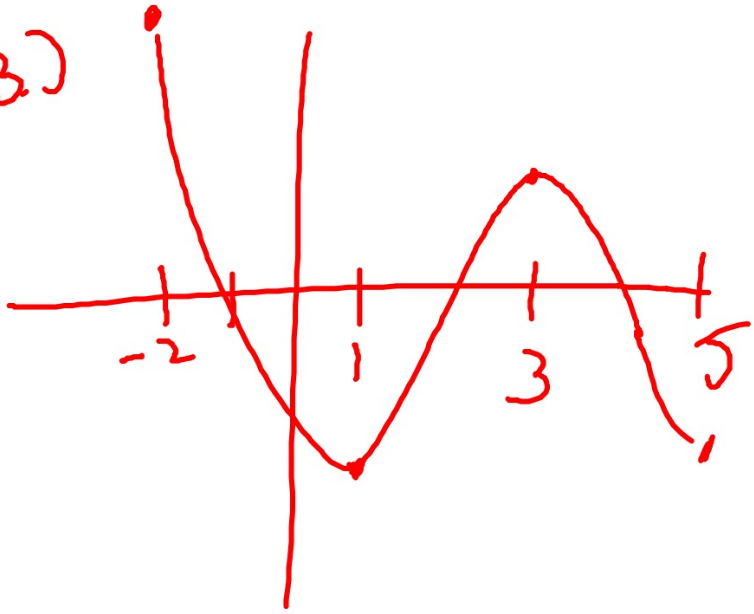
$\cos x + \sin x = 0$

$x = \frac{3\pi}{4} \to \cos x = -\sin x$

$-1 = \tan x$

x	y
0	0
$3\pi/4$	$e^{3\pi/4}\frac{\sqrt{2}}{2}$
π	0

(63)



3.2 Rolle's Theorem and the Mean Value Theorem

ex: List the critical numbers of f(x).

$$f(x) = x^{4/5}(x-5)^2$$

ex: Find the maximum and miniumum values of f(x) on the indicated interval.

$$f(x) = \sin x + \cos x, \quad \left[0, \frac{\pi}{2}\right]$$

THEOREM 3.4 The Mean Value Theorem

If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

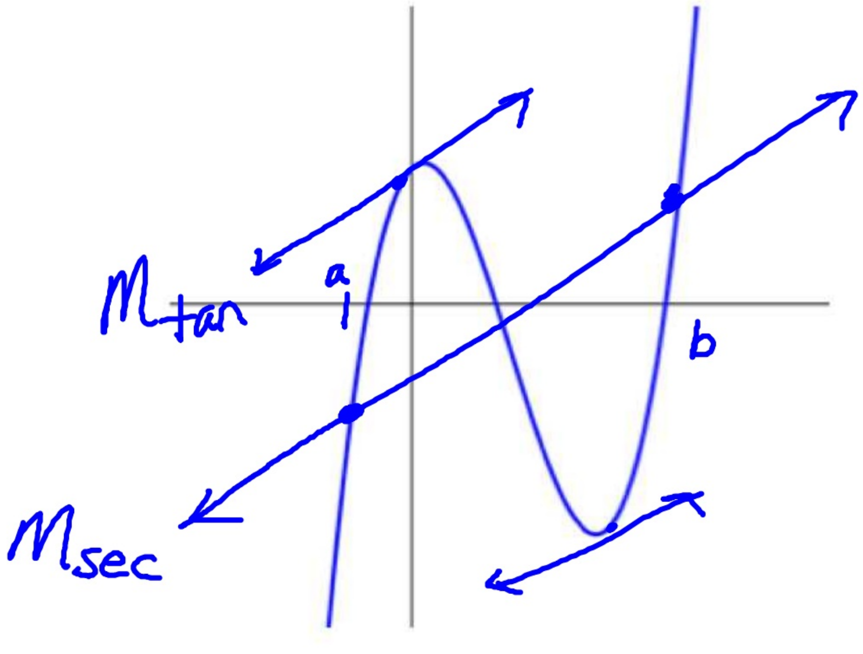
$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

MVT

slope at c

average rate of change on [a, b]

The Graphical Interpretation of the Mean Value Theorem.



ex: Determine the value(s) of c guaranteed by the conclusion of the MVT on the given interval, if possible.

a) $f(x) = 5 - \frac{4}{x}, \quad [1,4]$

Cont. ✓

diff ✓

$f'(x) = \frac{4}{x^2}$

$\frac{f(4) - f(1)}{4-1} = \frac{4-1}{3}$

$= 1$

$\frac{4}{c^2} = 1$

$c = \pm 2$

$\boxed{c = 2}$

ex: Determine the value(s) of c guaranteed by the conclusion of the MVT on the given interval, if possible.

b) $f(x) = \sqrt{x-4}, \quad [4,8]$

cont ✓

diff ✓

$$\frac{1}{2\sqrt{x-4}} = \frac{1}{2}$$

$$\sqrt{x-4} = 1$$

$$x = 5$$

$c = 5$

ex: Determine the value(s) of c guaranteed by the conclusion of the MVT on the given interval, if possible.

c) $f(x) = \frac{1}{x}, \quad [-1,1]$

MVT does not apply

not cont. at $x = 0$

ex: Determine the value(s) of c guaranteed by the conclusion of the MVT on the given interval, if possible.

d) $f(x) = |x|, \quad [-2,2]$

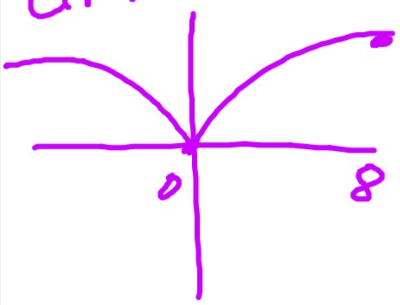
MVT does not apply

not. ~~diff~~ at $x=0$

e.) $f(x) = x^{2/3}$ $[0, 8]$

cont. ✓

diff. ✓



$$\frac{2}{3x^{1/3}} = \frac{1}{2}$$

$$3x^{1/3} = 4$$

$$x^{1/3} = \frac{4}{3}$$

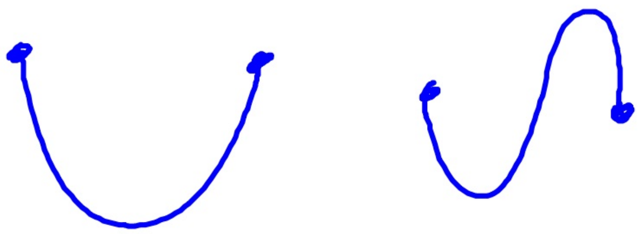
$$x = \frac{64}{27}$$

$$\boxed{c = \frac{64}{27}}$$

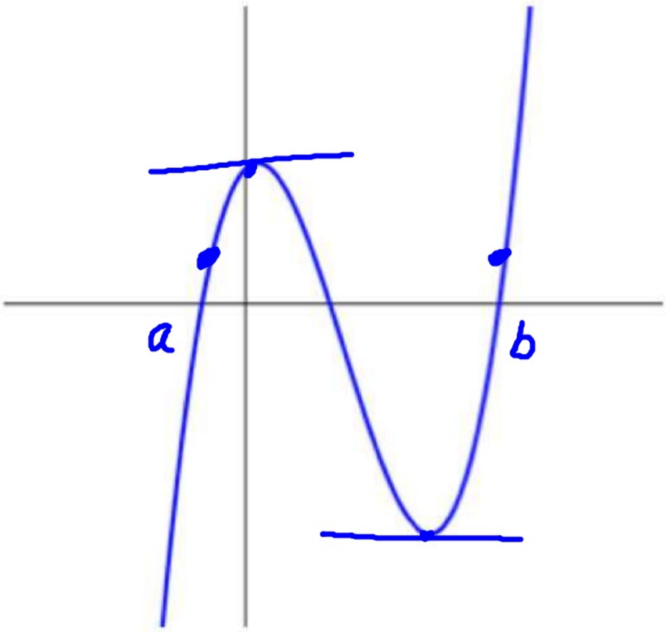
THEOREM 3.3 Rolle's Theorem

Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If $f(a) = f(b)$, then there is at least one number c in (a, b) such that

$$f'(c) = 0$$



The Graphical Interpretation of Rolle's Theorem.



ex: Determine the value(s) of c guaranteed by the conclusion of Rolle's Theorem on the given interval, if possible.

a) $f(x) = \cos\frac{x}{3}, \quad [0, 6\pi]$

cont ✓

diff ✓

$f(0) = f(6\pi)$

$1 = 1$ ✓

$f'(x) = 0$

$-\frac{1}{3}\sin\frac{x}{3} = 0$

$\sin\frac{x}{3} = 0$

$\frac{x}{3} = 0, \pi, 2\pi, 3\pi \ldots$

$x = 0, 3\pi, 6\pi, 9\pi$

$c = 3\pi$

ex: Determine the value(s) of c guaranteed by the conclusion of Rolle's Theorem on the given interval, if possible.

b) $f(x) = x, \quad [1,20]$

$f(1) \neq f(20)$

Rolle's Thm
does not apply

ex:

Let f be the function given by $f(x) = x^3 - 3x^2$. What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0,3]$?

~~(A) 0 only~~ (B) 2 only ~~(C) 3 only~~ ~~(D) 0 and 3~~ ~~(E) 2 and 3~~

$f'(x) = 3x^2 - 6x$

$0 = 3x(x-2)$

$\cancel{x=0}, \boxed{2}$

FR:

Let f be the function given by $f(x) = x^3 - 7x + 6$.

(a) Find the zeros of f. $\{1, 2, -3\}$

(b) Write an equation of the line tangent to the graph of f at $x = -1$. $y - 12 = -4(x+1)$

(c) Find the number c that satisfies the conclusion of the Mean Value Theorem for f on the closed interval $[1, 3]$.

$$3x^2 - 7 = 6$$

$$x = \pm\sqrt{\frac{13}{3}}$$

$$\boxed{c = \sqrt{\frac{13}{3}}}$$