

## 3.2 Rolle's Theorem and the Mean Value Theorem

ex: List the critical numbers of  $f(x)$ .

$$f(x) = x^{4/5} (x - 5)^2$$

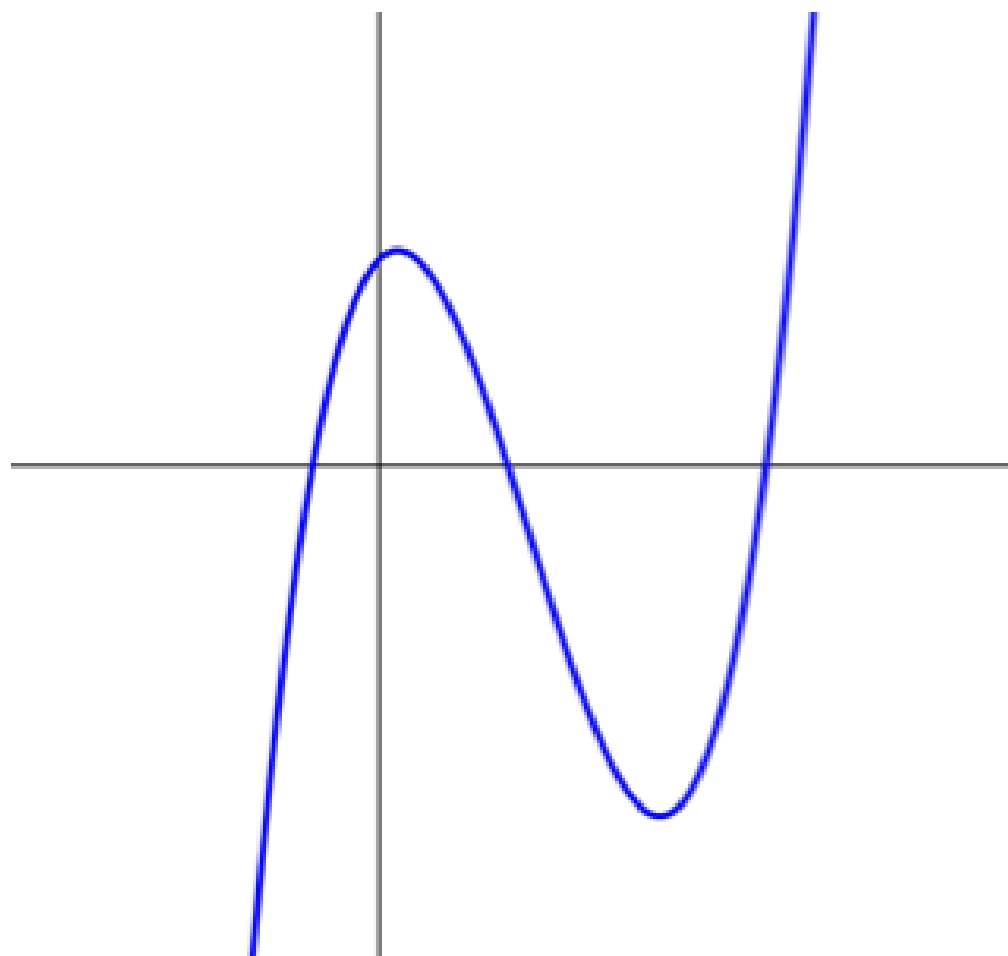
ex: Find the maximum and minimum values of  $f(x)$  on the indicated interval.

$$f(x) = \sin x + \cos x, \quad \left[0, \frac{\pi}{2}\right]$$

**THEOREM 3.4** The Mean Value Theorem

If  $f$  is continuous on the closed interval  $[a, b]$  and differentiable on the open interval  $(a, b)$ , then there exists a number  $c$  in  $(a, b)$  such that

# The Graphical Interpretation of the Mean Value Theorem.



ex: Determine the value(s) of  $c$  guaranteed by the conclusion of the MVT on the given interval, if possible.

a)  $f(x) = 5 - \frac{4}{x}, \quad [1, 4]$

ex: Determine the value(s) of  $c$  guaranteed by the conclusion of the MVT on the given interval, if possible.

b)  $f(x) = \sqrt{x-4}, \quad [4, 8]$

ex: Determine the value(s) of  $c$  guaranteed by the conclusion of the MVT on the given interval, if possible.

$$\textcircled{c} \quad f(x) = \frac{1}{x}, \quad [-1, 1]$$

ex: Determine the value(s) of  $c$  guaranteed by the conclusion of the MVT on the given interval, if possible.

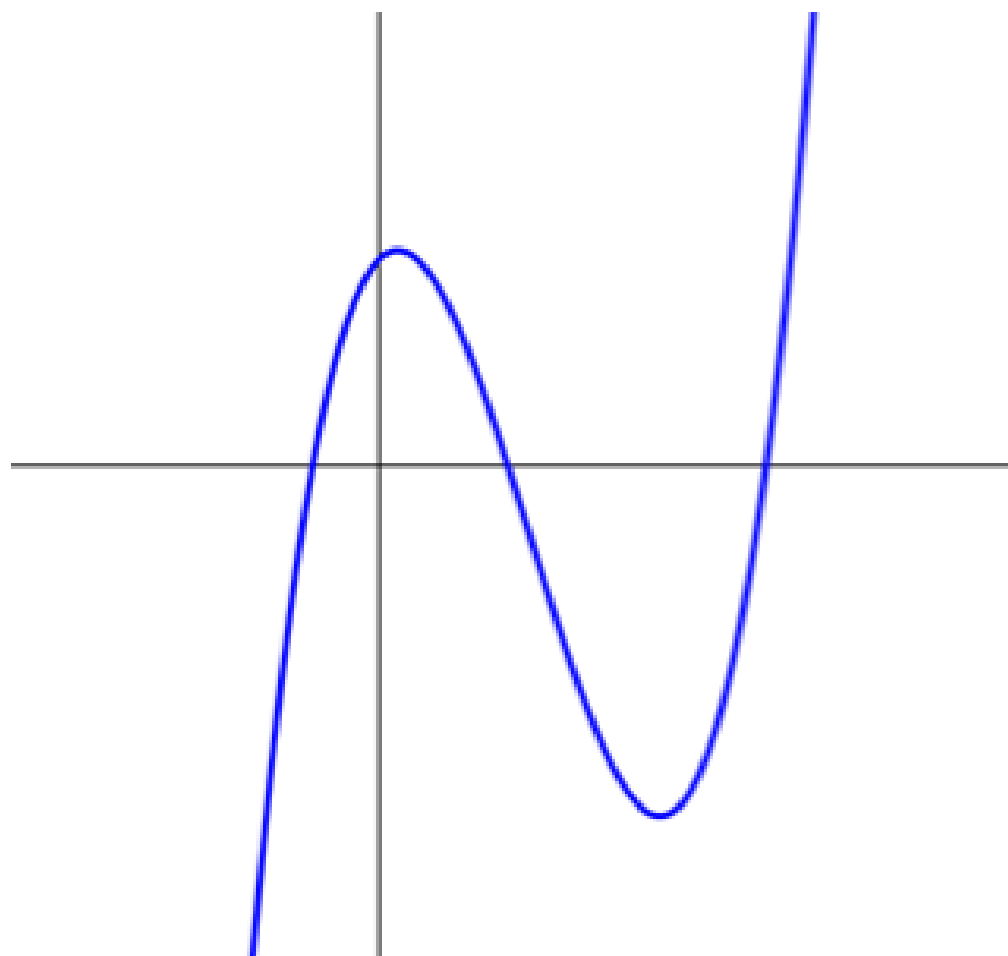
d)  $f(x) = |x|, \quad [-2, 2]$



### **THEOREM 3.3** Rolle's Theorem

Let  $f$  be continuous on the closed interval  $[a, b]$  and differentiable on the open interval  $(a, b)$ . If  $f(a) = f(b)$ , then there is at least one number  $c$  in  $(a, b)$  such that

# The Graphical Interpretation of Rolle's Theorem.



ex: Determine the value(s) of  $c$  guaranteed by the conclusion of Rolle's Theorem on the given interval, if possible.

a)  $f(x) = \cos \frac{x}{3}, \quad [0, 6\pi]$

ex: Determine the value(s) of  $c$  guaranteed by the conclusion of Rolle's Theorem on the given interval, if possible.

b)  $f(x) = x, \quad [1, 20]$

**ex:**

Let  $f$  be the function given by  $f(x) = x^3 - 3x^2$ . What are all values of  $c$  that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval  $[0, 3]$ ?

- (A) 0 only      (B) 2 only      (C) 3 only      (D) 0 and 3      (E) 2 and 3

**FR:**

Let  $f$  be the function given by  $f(x) = x^3 - 7x + 6$ .

- (a) Find the zeros of  $f$ .
- (b) Write an equation of the line tangent to the graph of  $f$  at  $x = -1$ .
- (c) Find the number  $c$  that satisfies the conclusion of the Mean Value Theorem for  $f$  on the closed interval  $[1, 3]$ .