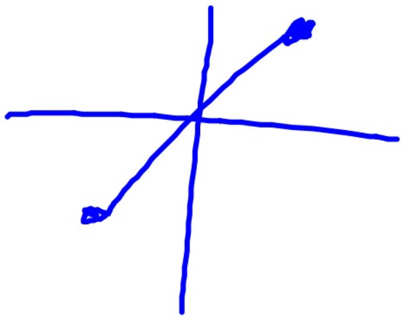
3.1 Extrema on an Interval

Definition of Extrema

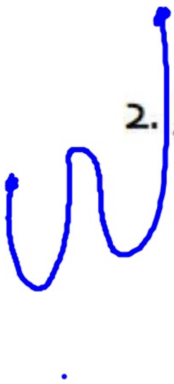
Let f be defined on an interval I containing c.

1. $f(c)$ is the **minimum of f on I** when $f(c) \leq f(x)$ for all x in I.
2. $f(c)$ is the **maximum of f on I** when $f(c) \geq f(x)$ for all x in I.



Types of Extrema

1. Absolute Extrema - the highest and lowest points on a curve (can occur ANYWHERE on a curve)

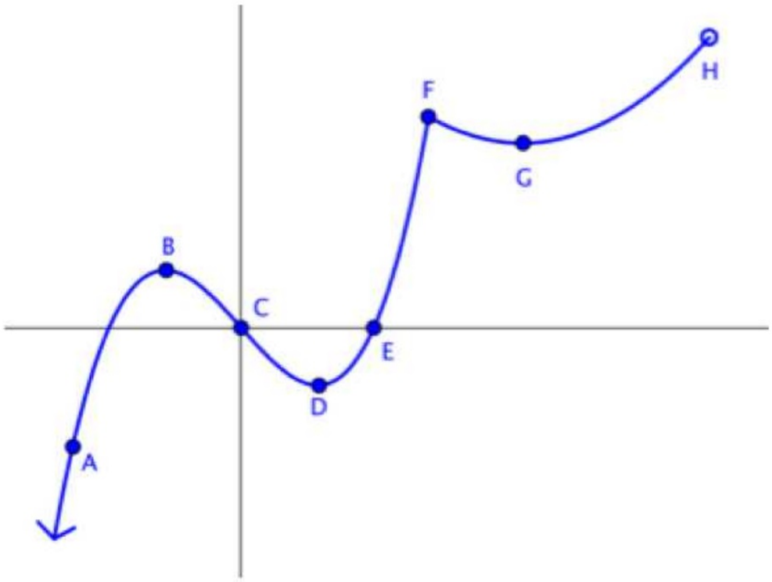


2. Relative Extrema - the highest and lowest points on a curve "in a neighborhood" (can occur ANYWHERE on an OPEN interval...no endpoints)
   (a.k.a. "Local Extrema)

When asked "where" does f(x) have extrema answer with an x-value.

When asked the extreme "value" of f(x) answer with a y-value.

ex:



At what point(s), if any, does f(x) have a(n)

a) Absolute Maximum none

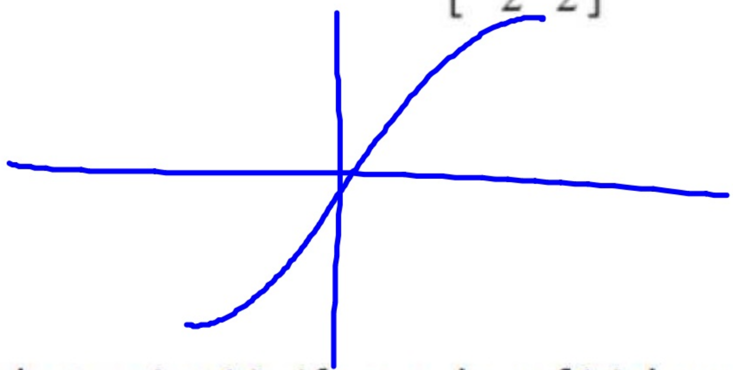
b)Absolute Minimum none

c) Relative Maximum B, F

d) Relative Minimum G, D

ex: $y = \sin x$

a) Sketch on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



At what point(s), if any, does f(x) have a(n)

b) Absolute Maximum $\left(\frac{\pi}{2}, 1\right)$

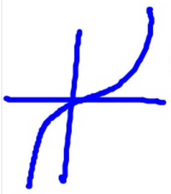
c)Absolute Minimum $\left(-\frac{\pi}{2}, -1\right)$

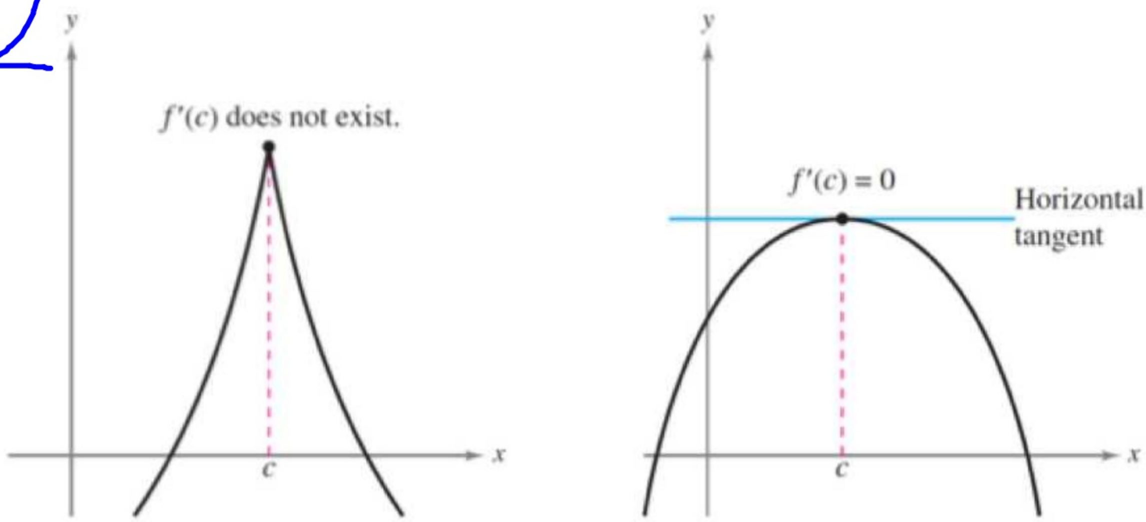
d) Relative Maximum none

e) Relative Minimum none

Definition of a Critical Number

Let f be defined at c. If $f'(c) = 0$ or if f is not differentiable at c, then c is a **critical number** of f.





ex: Find the critical numbers.

a) $f(x) = x^2 + 2x - 4$

$D: (-\infty, \infty)$

$f'(x) = 2x + 2$

$0 = 2x + 2$

$\boxed{-1 = x}$

$f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b$

$0 = 2ax + b$

$\frac{-b}{2a} = x$

ex: Find the critical numbers.

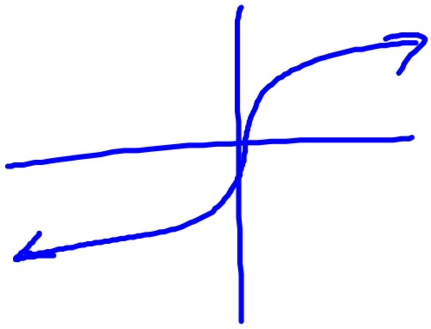
b) $f(x) = \sqrt[3]{x}$

$f'(x) = \frac{1}{3} x^{-2/3}$

$f'(x) = \frac{1}{3x^{2/3}}$

$3x^{2/3} = 0$

$\boxed{x = 0}$

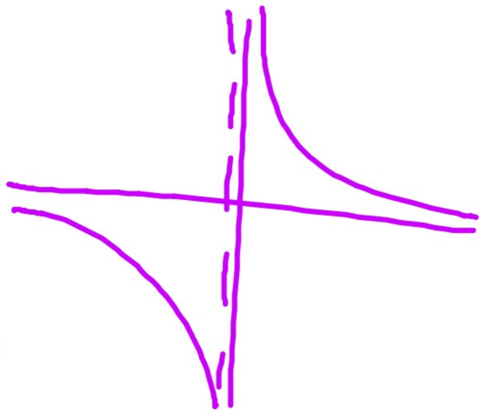


ex: Find the critical numbers.

c) $f(x) = \frac{1}{x}$

$D: x \neq 0$

$f'(x) = \frac{-1}{x^2}$

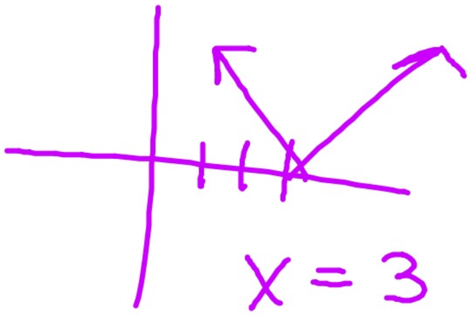


$x \neq 0$

no crit #

ex: Find the critical numbers.

d) $f(x) = |x - 3|$



$x = 3$

ex: Find the critical numbers.

e) $f(x) = 2x \ln x$ $D: (0, \infty)$

$f'(x) = 2x \cdot \frac{1}{x} + \ln x \cdot 2$

$0 = 2 + 2\ln x$

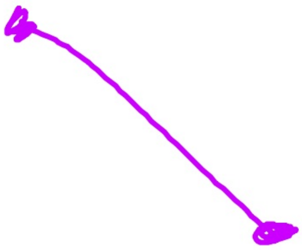
$0 = \cancel{2}(1 + \ln x)$

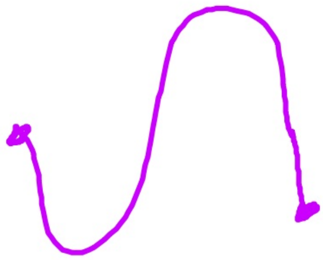
$-1 = \ln x$

$e^{-1} = x$

THEOREM 3.1 The Extreme Value Theorem EVT

If f is continuous on a closed interval $[a, b]$, then f has both a minimum and a maximum on the interval.





absolute extrema

ex: Find the ~~maximum and minimum values~~ of f(x) on the indicated interval.

a) $f(x) = 3x^4 - 4x^3, \quad [-1,2]$

$f'(x) = 12x^3 - 12x^2$

$0 = 12x^2(x-1)$

$x = 0, 1$

x	$f(x)$	(x,y)
-1	7	$(-1,7)$
0	0	$(0,0)$
1	-1	$(1,-1)$
2	16	$(2,16)$

abs. max $(2, 16)$

abs. min $(1, -1)$

ex: Find the maximum and minimum values of f(x) on the indicated interval.

b) $f(x) = 2\sin x - \cos 2x, \quad [0, \pi]$

$f'(x) = 2\cos x + 2\sin 2x$

$0 = 2\cos x + 2\sin 2x$

$0 = 2\cos x + 4\sin x\cos x$

$0 = 2\cos x(1 + 2\sin x)$

$2\cos x = 0 \quad 1 + 2\sin x = 0$

$x = \frac{\pi}{2} \quad \sin x = -\frac{1}{2}$

x	y
0	-1
π/2	3
π	-1

max valve: 3

min valve: -1

ex: What is the maximum acceleration on the interval [0,3] if the velocity is modeled by the equation

We need v''(t) to analyze what a(t) is doing

$$v(t) = t^3 - 3t^2 + 12t + 4$$

$$v'(t) = 3t^2 - 6t + 12$$

$$v''(t) = 6t - 6$$

$t = 1$

$$\boxed{\text{max accel: } 21}$$

t	$v'(t)$
0	12
1	9
3	21

Find the maximum and minimum value of $y = 2x - 3x^{2/3}$ on [-1,3]

$D: (-\infty, \infty)$

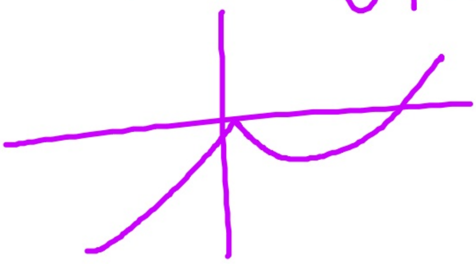
$y' = 2 - 2x^{-1/3}$

$y' = 2 - \frac{2}{x^{1/3}}$

$y' = \frac{2x^{1/3} - 2}{x^{1/3}}$

$x = 1$; $x = 0$

x	y
-1	-5 min value
0	0 max value
1	-1
3	$6 - 3\sqrt[3]{9}$



ex: Sketch a function with the given characteristics.

Relative minimum at $x = -1$

Critical number (but no extremum) at $x = 0$

Absolute maximum at $x = 2$

Absolute minimum at $x = 5$