45.) $V = \pi r^2 h = \pi \left(\frac{3}{2}\right)^2 h$

$\frac{dh}{dt} = -4 \text{ in/min}$ $\quad -1/3 \text{ ft/min}$

$V = \frac{9\pi}{4} h$

$d = 3$

$r = \frac{3}{2}$

$\frac{dV}{dt} = \frac{9\pi}{4} \frac{dh}{dt}$

$= \frac{9\pi}{4}\left(\frac{-1}{3}\right) = -\frac{3\pi}{4}$

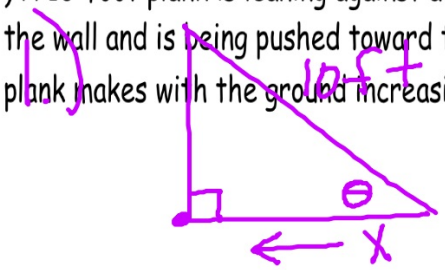
51.) $x^2 - y^2 = 36$

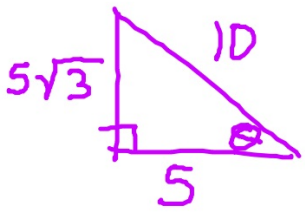
$$\frac{d}{dx}\left(\frac{dy}{dx} = \frac{x}{y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{y(1) - x\frac{dy}{dx}}{y^2}$$

$$= \frac{y - \frac{x^2}{y}}{y^2} = \frac{\frac{y^2 - x^2}{y}}{y^2} = \frac{y^2 - x^2}{y^3} = \frac{-36}{y^3}$$

) A 10-foot plank is leaning against a wall. If at a certain instant the bottom of the plank is 5 feet from the wall and is being pushed toward the wall at a rate of $\frac{1}{2}$ ft/sec, how fast is the acute angle that the plank makes with the ground increasing?





$\frac{dx}{dt} = \frac{1}{2}$ ft/sec

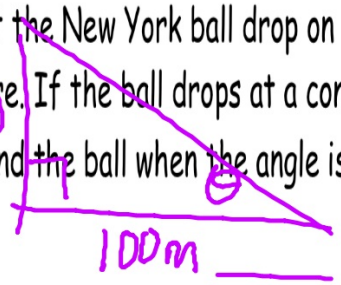
$\frac{d}{dt}\left(\cos\theta = \frac{x}{10}\right)$

$-\sin\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{10}\frac{dx}{dt}$

$+\left(\frac{\sqrt{3}}{2}\right)\frac{d\theta}{dt} = \frac{1}{10}\cdot\frac{+1}{2}$

$\frac{d\theta}{dt} = \frac{2}{\sqrt{3}}\cdot\frac{1}{20} = \frac{1}{10\sqrt{3}}$ rad/sec

2) You are looking at the New York ball drop on New Year's Eve at a distance of 100 m away from the base of the structure. If the ball drops at a constant rate of 2 m/s, what is the rate of change of the angle between you and the ball when the angle is $\pi/3$?



$\frac{dy}{dt} = -2$ m/sec

$\frac{d\theta}{dt} =$ ___ when $\theta = \frac{\pi}{3}$

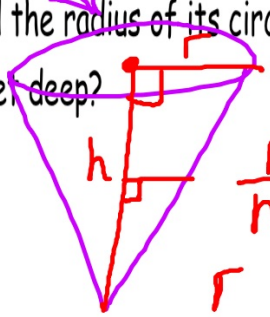
$\frac{d}{dt}\left(\tan\theta = \frac{y}{100}\right)$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{100}\frac{dy}{dt}$

$4 \cdot \frac{d\theta}{dt} = \frac{1}{100} \cdot (-2)$

$\frac{d\theta}{dt} = \frac{-1}{200}$ rad/sec

3) Water is pouring into a conical tank at the rate of 8 cubic feet per minute. If the height of the tank is 12 feet and the radius of its circular opening is 6 feet, how fast is the water level rising when the water is 4 feet deep?



$h = 12$
$r = 6$

$\frac{r}{h} = \frac{6}{12}$

$r = \frac{1}{2}h$

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{1}{2}h\right)^2 h$

$V = \frac{\pi}{12} h^3$

$\frac{dV}{dt} = 8 ft^3/min$

$\frac{dh}{dt} =$ ____ $h = 4ft$

$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$

$8 = 4\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{2}{\pi}$ ft/min