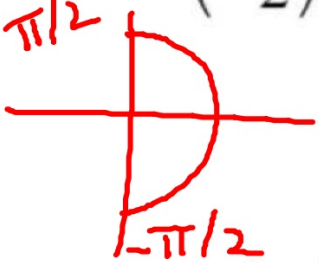
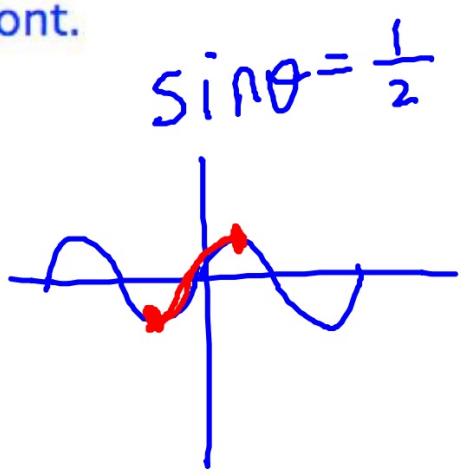
2.6 Derivatives of Inverse Functions Cont.

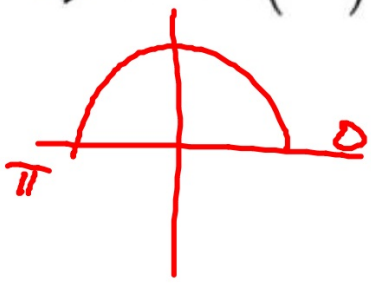
ex: Evaluate.

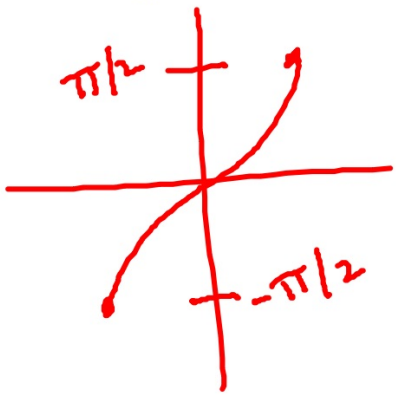
a) $\sin^{-1}\left(-\frac{1}{2}\right) = \frac{-\pi}{6}$

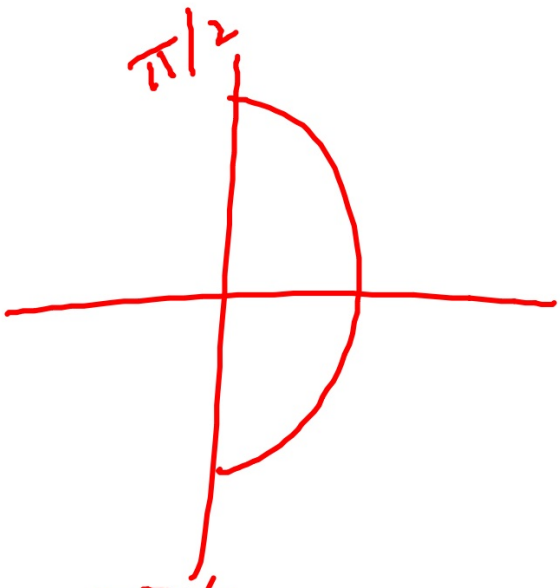




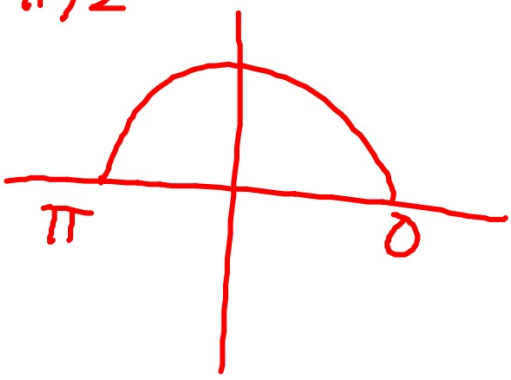
b) $\arccos(-1) = \pi$





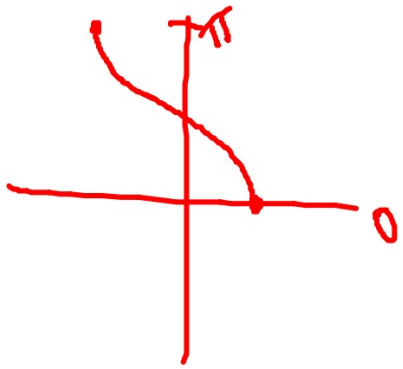


arcsinx
arccscx
arctanx

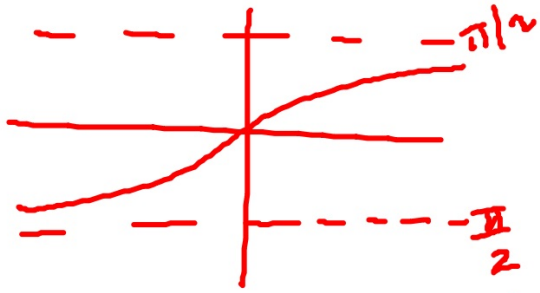


arccotx
arccosx
arcsecx

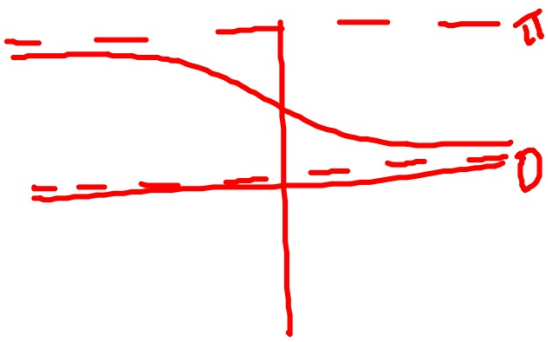
$\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$



$\arctan(-1) = \frac{-\pi}{4}$



$\operatorname{arccot}(1) = \frac{\pi}{4}$



Derivatives of Inverse Trigonometric Functions

THEOREM 2.18 Derivatives of Inverse Trigonometric Functions

Let u be a differentiable function of x.

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

ex: Find the derivative.

a) $y = \sin^{-1}(2x)$

$$y' = \frac{2}{\sqrt{1-4x^2}}$$

ex: Find the derivative.

b) $f(x) = \sec^{-1}\left(e^{7x}\right)$

$$f'(x) = \frac{7e^{7x}}{|e^{7x}|\sqrt{e^{14x}-1}}$$

ex: Find an equation of the tangent line to the graph of f at the given point.

$$y = \arctan\left(\frac{x}{2}\right), \quad x = -2$$

$\left(-2, -\frac{\pi}{4}\right)$

$$y' = \frac{\frac{1}{2}}{1 + \frac{x^2}{4}}$$

$$y' = \frac{2}{4 + x^2}$$

$y'(-2) = \frac{1}{4}$

$$y + \frac{\pi}{4} = \frac{1}{4}(x + 2)$$