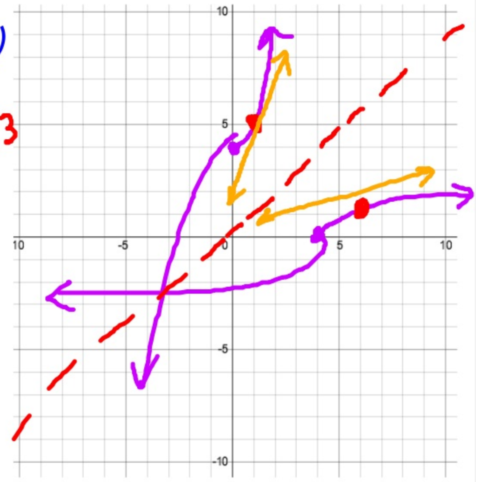
2.6: Derivative of Inverse Functions

Sketch. Compare the slopes at f(1) and $f^{-1}(5)$

$f(x) = x^3 + 4$ $\quad$ $f^{-1}(x) = \sqrt[3]{x-4}$

$f'(x) = 3x^2$ $\quad$ $(f^{-1})'(x) = \frac{1}{3}(x-4)^{-2/3}$

$f'(1) = 3$ $\quad$ $(f^{-1})'(5) = \frac{1}{3}$



1) $f(x) = 2x^3 + 3x$

Verify the function is 1:1 (monotonic: a function that is always increasing or always decreasing)

$f'(x) = 6x^2 + 3 > 0$

Find the derivative of the inverse at x = 5. $(f^{-1})'(5) = \underline{\quad \frac{1}{9} \quad}$

$5 = 2x^3 + 3x$

$1 = x$

$f'(x) = 6x^2 + 3$

$f'(1) = 9$

reciprocal

$f^{-1}: (5, 1)$

$f: (1, 5)$

THEOREM 5.9 THE DERIVATIVE OF AN INVERSE FUNCTION

Let f be a function that is differentiable on an interval I. If f has an inverse function g, then g is differentiable at any x for which $f'(g(x)) \neq 0$. Moreover,

$$g'(x) = \frac{1}{f'(g(x))}, \quad f'(g(x)) \neq 0.$$

2) $f(x) = x^3 - \frac{4}{x}$ Domain: $(0, \infty)$

f(x) and g(x) are inverses. Find g'(6) $= \frac{1}{13}$

$6 = x^3 - \frac{4}{x}$

$2 = x$

$f'(x) = 3x^2 + \frac{4}{x^2}$

$f'(2) = 13$

g: (6, 2)

f: (2, 6)

3. 18% answered correctly

If $f(x) = x^3 + x$ and $h(x)$ is the inverse of $f(x)$, then $h'(2)$ is

A) $\frac{1}{13}$ B) $\frac{1}{4}$ C) 1 D) 4 E) 13

$2 = x^3 + x$

$1 = x$

h : (2,)

f : (1, 2)

$f'(x) = 3x^2 + 1$

$f'(1) = 4$

4) Let f be a differentiable function with f(3) = 15, f'(3) = -8, f'(6) = -2, f(6) = 3. The function g is differentiable and $g(x) = f^{-1}(x)$. What is the value of g'(3)?

a) -1/8

b) -1/2

c) 1/6

d) 1/3

e) not possible

$f^{-1}(x)$

g: (3, 6)

f: (6, 3)

f'(6) = -2

5.

Use your calculator to determine the derivative of the inverse function of $f(x)$ at $x = 4$ where $f(x) = x^5 + x^3 + 2x - 2$.

6.

Suppose f is a one-to-one function, which is differentiable for all real numbers x. The table below gives some of the values of $f(x)$ and $f'(x)$:

x	$f(x)$	$f'(x)$
1	2	$\frac{7}{6}$
2	3	$\frac{7}{6}$
3	5	$\frac{19}{6}$
4	10	$\frac{43}{6}$

(a) Write an equation of the tangent line, T_1, to the function $f(x)$ at $x = 3$.

(b) Write an equation of the normal line, N_1, to the function $f(x)$ at $x = 3$.

(c) Write an equation of the tangent line, T_2, to the function $f^{-1}(x)$ at $x = 3$.

$f'(2) = \frac{7}{6}$

$y - 2 = \frac{6}{7}(x - 3)$

$f^{-1}: (3, 2)$

$f: (2, 3)$

7.

The function used in Problems 4 and 5 is $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{19}{6}x$.

(a) $f'(0) =$ $\underline{\quad 19/6 \quad}$.

(b) If $g(x) = f^{-1}(x)$, then $g'(0) =$ $\underline{\quad 6/19 \quad}$.

(c) Write an equation for the normal line to $g(x)$ at $x = 0$.

$$y = -\frac{19}{6}x$$

$g : (0, 0)$

$f : (0, 0)$

AP Question. Mean Score 0.95

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

$h(1) = 3$

$h(3) = -7$

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

(c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)dt$. Find the value $w'(3)$.

(d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

$g'(1) = 5$

$g^{-1}: (2, 1)$

$g: (1, 2)$

$y - 1 = \frac{1}{5}(x - 2)$

7.7 Indeterminate Forms and L'Hopital's Rule

- 7 Indeterminate Forms

$$\boxed{\frac{0}{0}, \quad \frac{\infty}{\infty}, \quad 1^{\infty}, \quad 0^{0}, \quad \infty^{0}, \quad \infty \cdot 0, \quad \infty - \infty}$$

NOT INDETERMINATE FORMS:

$$\infty \cdot \infty$$

$$\infty + \infty$$

L'Hopital's Rule

THEOREM 7.4 L'Hôpital's Rule

Let f and g be functions that are differentiable on an open interval (a, b) containing c, except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b), except possibly at c itself. If the limit of $f(x)/g(x)$ as x approaches c produces the indeterminate form $0/0$, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

provided the limit on the right exists (or is infinite). This result also applies when the limit of $f(x)/g(x)$ as x approaches c produces any one of the indeterminate forms ∞/∞, $(-\infty)/\infty$, $\infty/(-\infty)$, or $(-\infty)/(-\infty)$.

ex: Find the limit or explain why it does not exist.

a) $\lim_{x\to 0}\frac{e^{2x}-1}{x} \overset{LHOP}{=} \lim_{x\to 0}\frac{2e^{2x}}{1} = 2$

$\frac{0}{0}$

ex: Find the limit or explain why it does not exist.

b) $\lim_{x\to\infty} \frac{\ln x}{x} \overset{\text{LHop}}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$

$\frac{\infty}{\infty}$

$$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} \stackrel{LHop}{=} \lim_{x\to 3} \frac{\frac{1}{2\sqrt{x+1}}}{1} = \frac{1}{4}$$

$$\frac{0}{0}$$

$$\lim_{x\to 0} \frac{\sin x}{x} \stackrel{LHop}{=} \lim_{x\to 0} \frac{\cos x}{1}$$

$$= 1$$