2.5 Implicit Differentiation

Implicit Form	*Explicit Form*
$xy = 1$	$y = \frac{1}{x} = x^{-1}$

explicit

ex: If $x^2 - y^2 = 16$ find dy/dx.

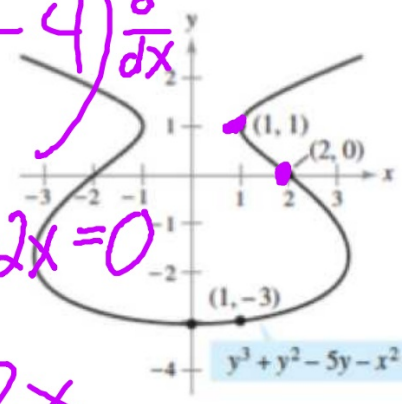
$y = \pm\sqrt{x^2-16}$

$y' = \pm\frac{1}{2}(x^2-16)^{-1/2}\cdot 2x$

$y' = \pm\frac{x}{\sqrt{x^2-16}}$

Implicit differentiation is nessecary to derive equations that can only be expressed implicitly.

$$\left(y^3 + y^2 - 5y - x^2 = -4\right)\frac{d}{dx}$$



$$3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5\frac{dy}{dx} - 2x = 0$$

$$\frac{dy}{dx}\left(3y^2 + 2y - 5\right) = 2x$$

$$\frac{dy}{dx} = \frac{2x}{3y^2 + 2y - 5}$$

$$\left.\frac{dy}{dx}\right|_{(2,0)} = \frac{4}{-5}$$

$$\frac{d^2y}{dx^2}$$

ex: If $x^2 - y^2 = 16$ find dy/dx implicitly.

$$\frac{d}{dx}\left(x^2 - y^2 = 16\right)$$

$$2x \cdot \frac{dx}{dx} - 2y \cdot \frac{dy}{dx} = 0$$

$$+2y\frac{dy}{dx} = +2x$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$y = \pm\sqrt{x^2-16}$

$$\frac{dy}{dx} = \pm\frac{x}{\sqrt{x^2-16}}$$

$$= \frac{x}{y}$$

ex: Find dy/dx.

a) $\frac{d}{dx}\left(x^2 - 2y^3 + 4y = 2\right)$

$2x - 6y^2\frac{dy}{dx} + 4\frac{dy}{dx} = 0$

$\frac{dy}{dx}\left(-6y^2 + 4\right) = -2x$

$\frac{dy}{dx} = \frac{-2x}{-6y^2+4} = \frac{2x}{6y^2-4} = \frac{x}{3y^2-2} = \frac{-x}{-3y^2+2}$

ex: Find dy/dx.

b) $\left(x^2y - 2\cos 3x = 3\right)\frac{d}{dx}$

$$x^2 \cdot \frac{dy}{dx} + y \cdot 2x + 6\sin 3x = 0$$

$$x^2\frac{dy}{dx} = -2xy - 6\sin 3x$$

$$\frac{dy}{dx} = \frac{-2xy - 6\sin 3x}{x^2}$$

ex: Find dy/dx.

e) $y = \sin(xy)$

ex: $x^2 - xy + y^2 = 7$

a) Find dy/dx.

b) Find the slope at the point (-1, 2).

c) Write an equation of the tangent line to the graph at the point (-1, 2).

d) Write an equation of the normal line to the graph at the point (-1, 2).