4b.) $s(t) = -16t^2 + 96t + 80$

$$0 = -16(t^2 - 6t - 5)$$

$$t = \frac{6 \pm \sqrt{36 - 4(1)(-5)}}{2(1)}$$

6.742 sec

$$= \frac{6 \pm \sqrt{56}}{2} = \frac{6 + \sqrt{56}}{2} \qquad \cancel{\frac{6 - \sqrt{56}}{2}}$$

4c.) $s'(t) = -32t + 96$

$s'(6.742) = -119.744$ ft/sec

2.5 Implicit Differentiation

Implicit Form	*Explicit Form*
$xy = 1$	$y = \frac{1}{x} = x^{-1}$

ex: If $x^2 - y^2 = 16$ find dy/dx.

$$y = \pm\sqrt{x^2 - 16}$$

$$y' = \pm \frac{1}{2}(x^2-16)^{-1/2} \cdot 2x$$

$$= \pm \frac{x}{\sqrt{x^2-16}}$$

$$\frac{d}{dx}\left(x^2 - y^2 = 16\right)$$

$$2x \cdot \frac{dx}{dx} - 2y \cdot \frac{dy}{dx} = 0$$

$$2x - 2y\frac{dy}{dx} = 0$$

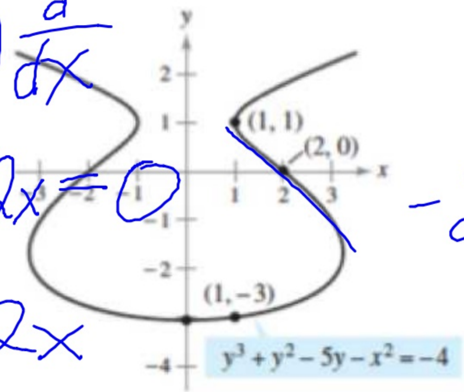
$$x = y\frac{dy}{dy}$$

$$\frac{x}{y} = \frac{dy}{dx}$$

$$\frac{x}{\pm\sqrt{x^2-16}}$$

Implicit differentiation is nessecary to derive equations that can only be expressed implicitly.

$$\frac{d}{dx}\left(y^3+y^2-5y-x^2=-4\right)$$



$$\underbrace{3y^2\cdot\frac{dy}{dx}+2y\frac{dy}{dx}-5\frac{dy}{dx}}-2x=0$$

$$-2x\frac{\cancel{dx}}{\cancel{dx}}\ (1)$$

$$\frac{dy}{dx}\left(3y^2+2y-5\right)=2x$$

$$\frac{dy}{dx}=\frac{2x}{3y^2+2y-5}$$

$$\left.\frac{dy}{dx}\right|_{(2,0)}=\frac{4}{-5}$$

ex: Find dy/dx.

a) $\left(x^2 - 2y^3 + 4y = 2x\right)\frac{d}{dx}$

$$2x - 6y^2\frac{dy}{dx} + 4\frac{dy}{dx} = 2$$

$$-6y^2\frac{dy}{dx} + 4\frac{dy}{dx} = 2 - 2x$$

$$\frac{dy}{dx}\left(-6y^2 + 4\right) = 2 - 2x$$

$$\frac{dy}{dx} = \frac{2-2x}{-6y^2+4} = \frac{\cancel{2}(1-x)}{\cancel{2}(-3y^2+2)} = \frac{-1(x-1)}{-1(3y^2-2)}$$

ex: Find dy/dx.

$\frac{d}{dx}$ b) $\left(x^2y - 2\cos 3x = 3\right)$

$x^2 \cdot \frac{dy}{dx} + y \cdot 2x + 6\sin 3x = 0$

$x^2 \frac{dy}{dx} = -2xy - 6\sin 3x$

$\frac{dy}{dx} = \frac{-2xy - 6\sin 3x}{x^2}$

ex: Find dy/dx.

e) $\frac{d}{dx}\left(y=\sin(xy)\right)$

$$\frac{dy}{dx}=\cos(xy)\cdot\left(x\frac{dy}{dx}+y\right)$$

$$\frac{dy}{dx}=\cos(xy)x\frac{dy}{dx}+y\cos(xy)$$

$$\frac{dy}{dx}-\cos(xy)x\frac{dy}{dx}=y\cos(xy)$$

$$\frac{dy}{dx}\left(1-x\cos(xy)\right)=y\cos(xy)$$

$$\frac{dy}{dx}=\frac{y\cos(xy)}{1-x\cos(xy)}$$

ex: If $x^2 + y^2 = 1$ find d^2y/dx^2.

$$\frac{d^2y}{(dx)^2}$$

$$\frac{d}{dx}\left(\frac{dy}{dx} = \frac{-x}{y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\frac{dy}{dx}}{y^2} = \frac{\left(-y + x\left(-\frac{x}{y}\right)\right)y}{(y^2)y}$$

$$= \frac{-y^2 - x^2}{y^3} = -\frac{(y^2 + x^2)}{y^3} = \frac{-1}{y^3}$$

given $x^2+y^2=1$

Show that

$$\frac{d^2y}{dx^2} = \frac{-1}{y^3}$$

ex: $x^2 - xy + y^2 = 7$

$x^2 - (x \cdot y) + y^2 = 7$

a) Find dy/dx.

$2x - xy' - y + 2yy' = 0$

$y' = \frac{-2x+y}{2y-x}$

b) Find the slope at the point (-1, 2).

$\left.\frac{dy}{dx}\right|_{(-1,2)} = \frac{4}{5}$

c) Write an equation of the tangent line to the graph at the point (-1, 2).

$y - 2 = \frac{4}{5}(x+1)$

d) Write an equation of the normal line to the graph at the point (-1, 2).

$y - 2 = -\frac{5}{4}(x+1)$

ex: $4x^2 + y^2 - 8x + 4y + 4 = 0$ $\quad \frac{dy}{dx} = \frac{8-8x}{2y+4}$

a) Find the points, if any, at which the equation has a horizontal tangent line. set num. of $\frac{dy}{dx}$ equal to 0

$(1, 0)(1, -4)$

b) Find the points, if any, at which the equation has a vertical tangent line. Set den. of $\frac{dy}{dx}$ equal to 0

$(0, -2)(2, -2)$