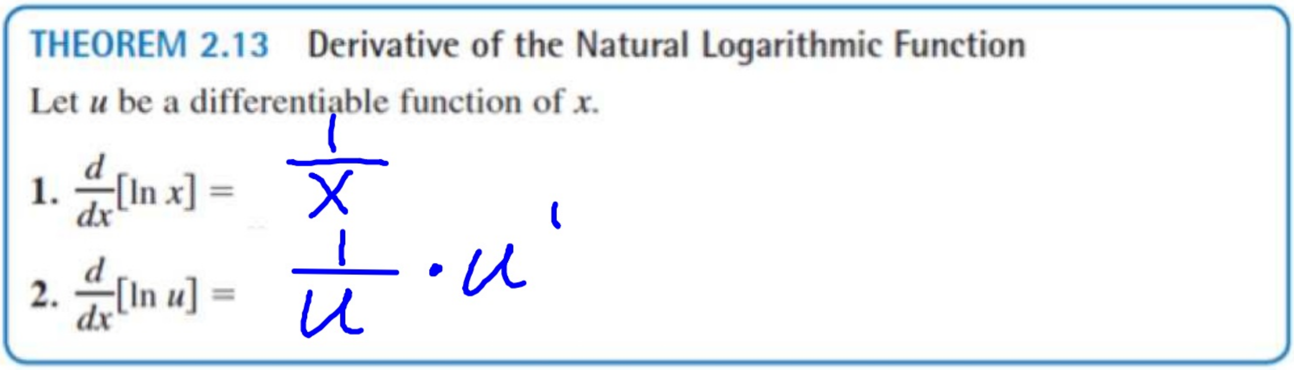
2.4 Chain Rule Cont.

ex: Differentiate.

a) $y = \csc\left(7x\right)$

b) $y = \dfrac{6}{\left(3x^2 - 5\right)^5}$



THEOREM 2.13 **Derivative of the Natural Logarithmic Function**

Let u be a differentiable function of x.

1. $\frac{d}{dx}[\ln x] = \frac{1}{x}$

2. $\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot u'$

ex: Differentiate.

a) $y = \ln(2x)$ $\quad y' = \frac{1}{2x} \cdot 2 = \frac{1}{x}$

ex: Differentiate.

b) $y = x \ln x$

$$y' = x \cdot \frac{1}{x} + \ln x \cdot 1$$

$$y' = 1 + \ln x$$

$$1 + \ln x = 0$$

$$e^{\ln x} = e^{-1}$$

$$x = \frac{1}{e}$$

ex: Differentiate.

c) $y = \ln\sqrt{x^2 + 8e^x}$

$$y = \frac{1}{2}\ln(x^2 + 8e^x)$$

$$y' = \frac{1}{2} \cdot \frac{1}{x^2+8e^x} \cdot (2x + 8e^x)$$

$$y' = \frac{x+4e^x}{x^2+8e^x}$$

ex: Differentiate.

d) $f(x) =$ ~~$\ln(x\sin x)$~~

$$\ln\left(\frac{x^2+1}{x-1}\right)^4$$

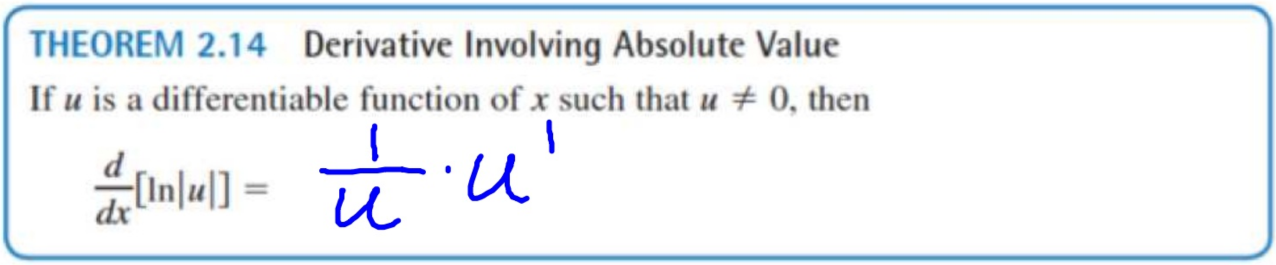
$$f(x) = 4\left(\ln(x^2+1) - \ln(x-1)\right)$$

$$(\ln x)^2$$

$$f'(x) = 4\left(\frac{1 \cdot 2x}{x^2+1} - \frac{1}{x-1}\right)$$

$$\ln x^2$$

$$\frac{8x}{x^2+1} - \frac{4}{x-1}$$



THEOREM 2.14 Derivative Involving Absolute Value

If u is a differentiable function of x such that $u \neq 0$, then

$$\frac{d}{dx}[\ln|u|] = \frac{1}{u} \cdot u'$$

ex: Differentiate.

a) $f(x) = \ln|\csc x|$

$$f'(x) = \frac{1}{\csc x}(-\csc x \cot x)$$

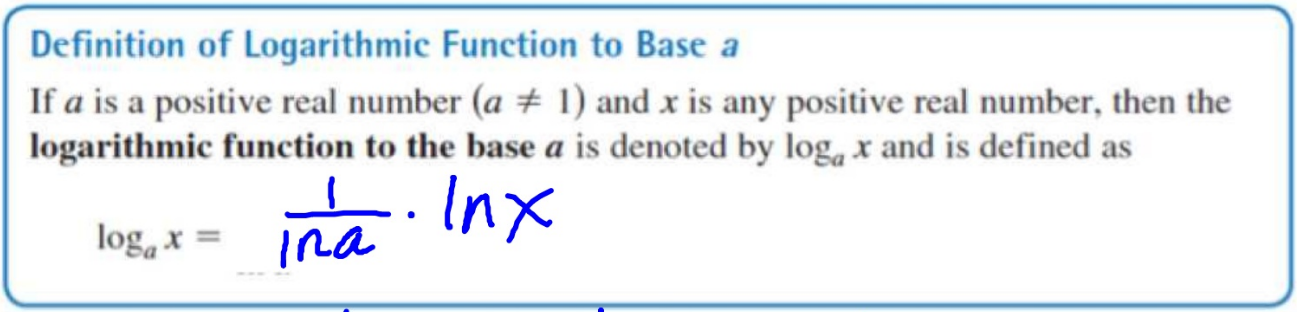
$$= -\cot x$$

ex: Differentiate.

b) $f\left(x\right)=\ln\left|\frac{\cos x-2}{e^{x}}\right|$

ex: Differentiate.

c) $y = \left|\ln x\right|$



Definition of Logarithmic Function to Base a

If a is a positive real number ($a \neq 1$) and x is any positive real number, then the **logarithmic function to the base a** is denoted by $\log_a x$ and is defined as

$$\log_a x = \frac{1}{\ln a} \cdot \ln x$$

$$\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a} = \frac{1}{\ln a} \cdot \ln x$$

$$\frac{d}{dx}\left[\log_a x\right] = \frac{1}{\ln a} \cdot \frac{1}{x}$$

$$\frac{d}{dx}\left[\log_a u\right] = \frac{1}{\ln a} \cdot \frac{1}{u} \cdot u'$$

ex: Differentiate.

a) $y = \log_7 x$

$$y' = \frac{1}{\ln 7} \cdot \frac{1}{x}$$

$$= \frac{1}{x \ln 7}$$

$$\frac{1}{(\ln 7)x}$$

ex: Differentiate.

b) $y = \log_6 \frac{x\sqrt{x-2}}{5}$

Derivative of the Exponential Function

$\ln e = 1$

$\ln 1 = 0$

$e^{\ln x} = x$

$2^{\log_2 x} = x$

Let u be a differentiable function of x.

$$\frac{d}{dx}\left[a^x\right] = \ln a \cdot a^x \qquad \frac{d}{dx}\left[e^x\right] = e^x$$

$$\frac{d}{dx}\left[a^u\right] = \ln a \cdot a^u \cdot u' \qquad \frac{d}{dx}\left[e^u\right] = e^u \cdot u'$$

ex: Differentiate.

a) $y = 3^x$

$y' = \ln 3 \cdot 3^x$

b) $y = e^{5x}$

$y' = 5e^{5x}$

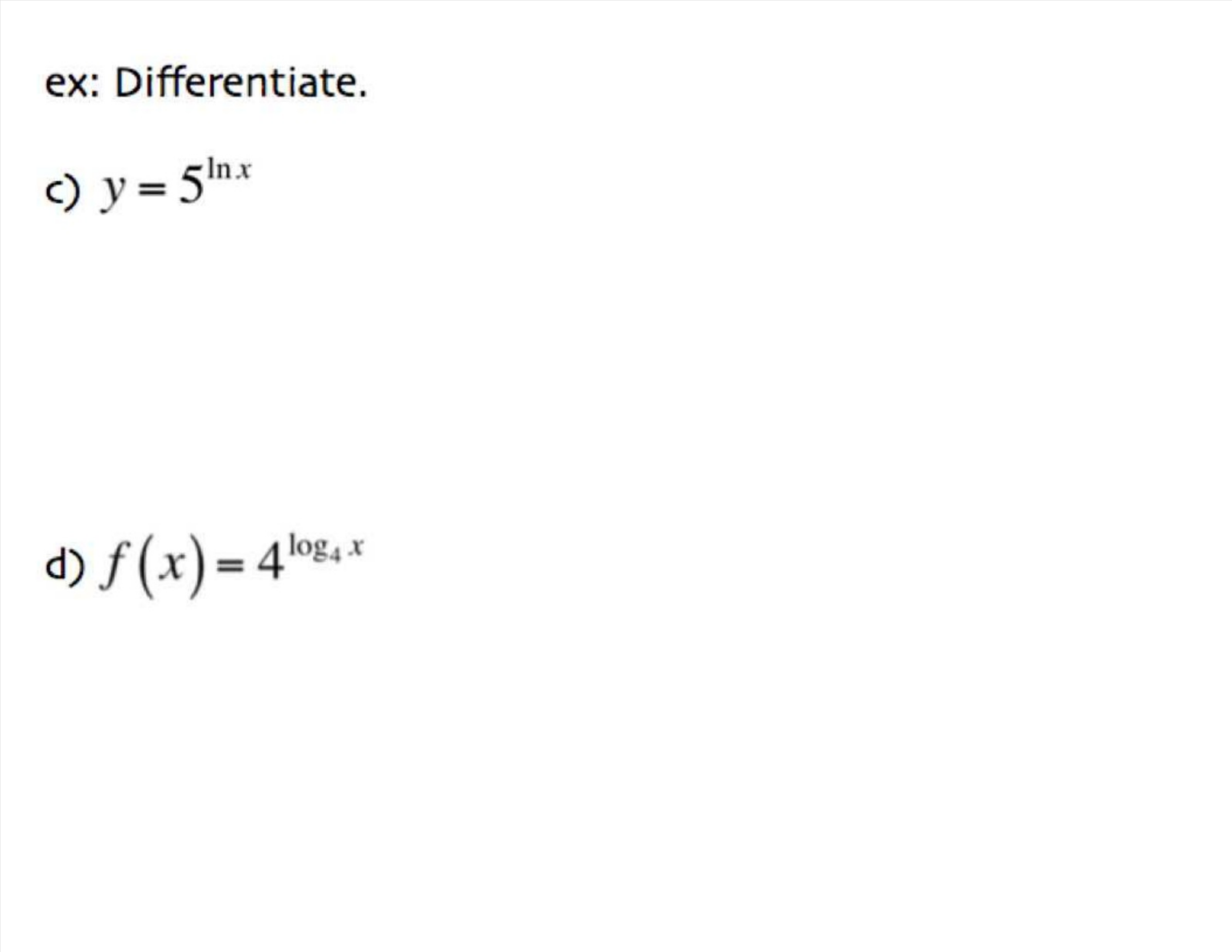
c.) $y = 5^{\ln x}$

$y' = 5^{\ln x} \cdot \ln 5 \cdot \frac{1}{x}$

d.) $f(x) = 4^{\log_4 x}$

$= x$

$f'(x) = 1$



ex: Differentiate.

c) $y = 5^{\ln x}$

d) $f\left(x\right) = 4^{\log_4 x}$

ex: Differentiate.

e) $y = \frac{3}{e^x} = 3e^{-x}$

$$y' = 3e^{-x}(-1)$$
$$= -3e^{-x}$$

$$\frac{e^x(1-e^x)+e^x(e^x+1)}{(e^x+1)(1-e^x)}$$

$$\frac{e^x \cancel{-e^{2x}} \cancel{+e^{2x}} + e^x}{(e^x+1)(1-e^x)} = \frac{2e^x}{(1+e^x)(1-e^x}$$

f) $y = \ln\left(\frac{1+e^x}{1-e^x}\right) = \ln(1+e^x) - \ln(1-e^x)$

$$y' = \frac{e^x}{1+e^x} - \frac{-e^x}{1-e^x} = \frac{e^x}{1+e^x} + \frac{e^x}{1-e^x}$$

ex: Differentiate.

g) $y = e^x \ln x$

$$y' = e^x \frac{1}{x} + \ln x e^x$$

$$= e^x\left(\frac{1}{x} + \ln x\right)$$

h) $f(x) = \ln e^{777x}$

$$= 777x$$

$$f'(x) = 777$$

ex: Write the equation of the tangent line to

$y = e^{4x}$ at $x = \ln 2$. $(\ln 2, 16)$

$$y' = 4e^{4x}$$

$$y'(\ln 2) = 4e^{4\ln 2}$$

$$= 4(16)$$

$$= 64$$

$$\boxed{y - 16 = 64(x - \ln 2)}$$

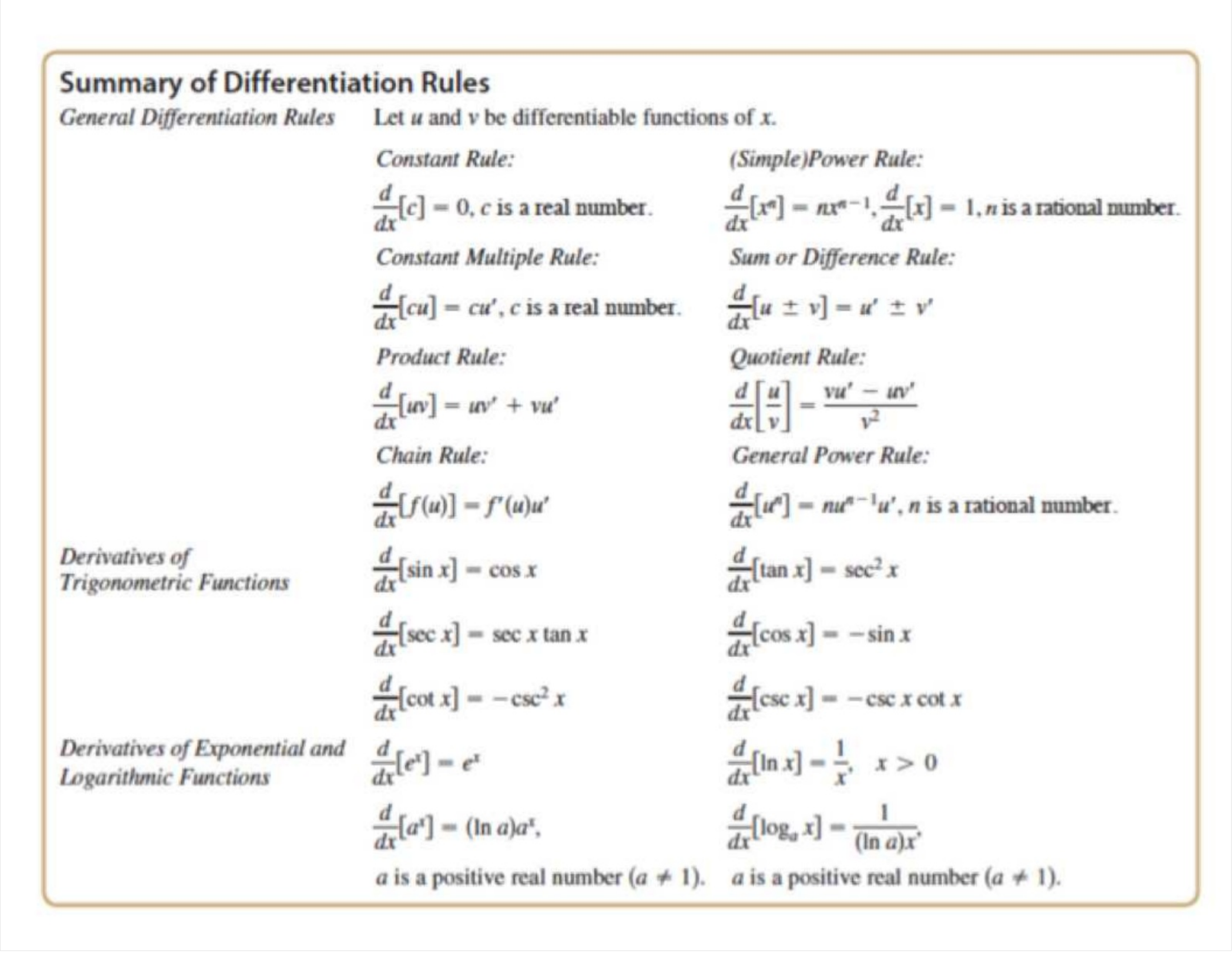
ex: Write an equation of the ~~tangent~~ normal line to $y = \frac{\ln x}{4x} = \frac{1}{4}\ln x \cdot x^{-1}$
at x=1. $(1, 0)$

$$y' = \frac{4x \cdot \frac{1}{x} - \ln x \cdot 4}{16x^2}$$

$$= \frac{4 - 4\ln x}{16x^2}$$

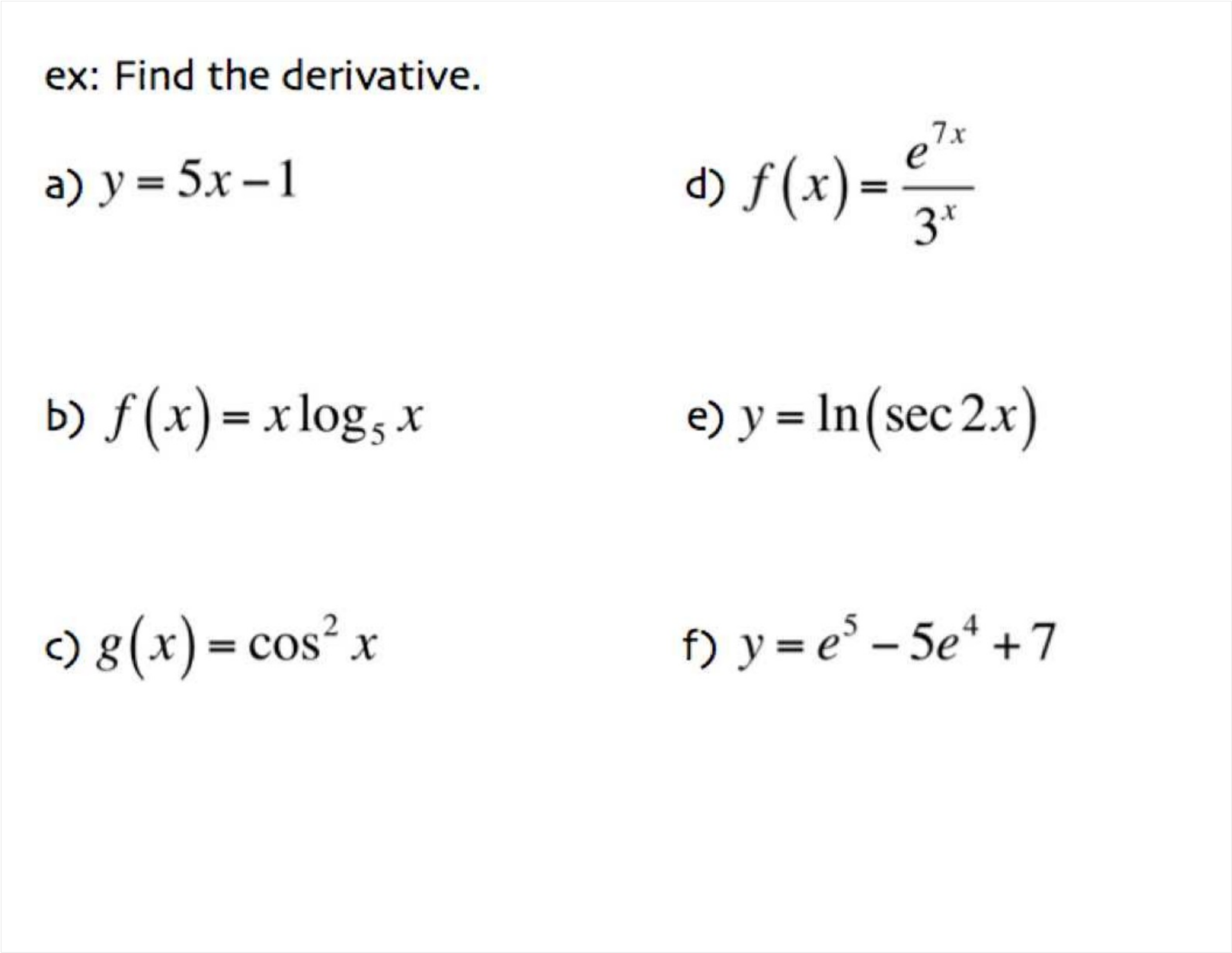
$$\frac{4}{16} = \frac{1}{4} \xrightarrow{\text{normal}} \boxed{y = -4(x-1)}$$

ex: Write an equation of the normal line to $y = \frac{\ln x}{4x}$
at x=1.



Summary of Differentiation Rules

General Differentiation Rules	Let u and v be differentiable functions of x.	
	Constant Rule:	*(Simple)Power Rule:*
	$\frac{d}{dx}[c] = 0$, c is a real number.	$\frac{d}{dx}[x^n] = nx^{n-1}, \frac{d}{dx}[x] = 1$, n is a rational number.
	Constant Multiple Rule:	*Sum or Difference Rule:*
	$\frac{d}{dx}[cu] = cu'$, c is a real number.	$\frac{d}{dx}[u \pm v] = u' \pm v'$
	Product Rule:	*Quotient Rule:*
	$\frac{d}{dx}[uv] = uv' + vu'$	$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
	Chain Rule:	*General Power Rule:*
	$\frac{d}{dx}[f(u)] = f'(u)u'$	$\frac{d}{dx}[u^n] = nu^{n-1}u'$, n is a rational number.
Derivatives of Trigonometric Functions	$\frac{d}{dx}[\sin x] = \cos x$	$\frac{d}{dx}[\tan x] = \sec^2 x$
	$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\frac{d}{dx}[\cos x] = -\sin x$
	$\frac{d}{dx}[\cot x] = -\csc^2 x$	$\frac{d}{dx}[\csc x] = -\csc x \cot x$
Derivatives of Exponential and Logarithmic Functions	$\frac{d}{dx}[e^x] = e^x$	$\frac{d}{dx}[\ln x] = \frac{1}{x}, \quad x > 0$
	$\frac{d}{dx}[a^x] = (\ln a)a^x$,	$\frac{d}{dx}[\log_a x] = \frac{1}{(\ln a)x}$,
	a is a positive real number ($a \neq 1$).	a is a positive real number ($a \neq 1$).



ex: Find the derivative.

a) $y = 5x - 1$

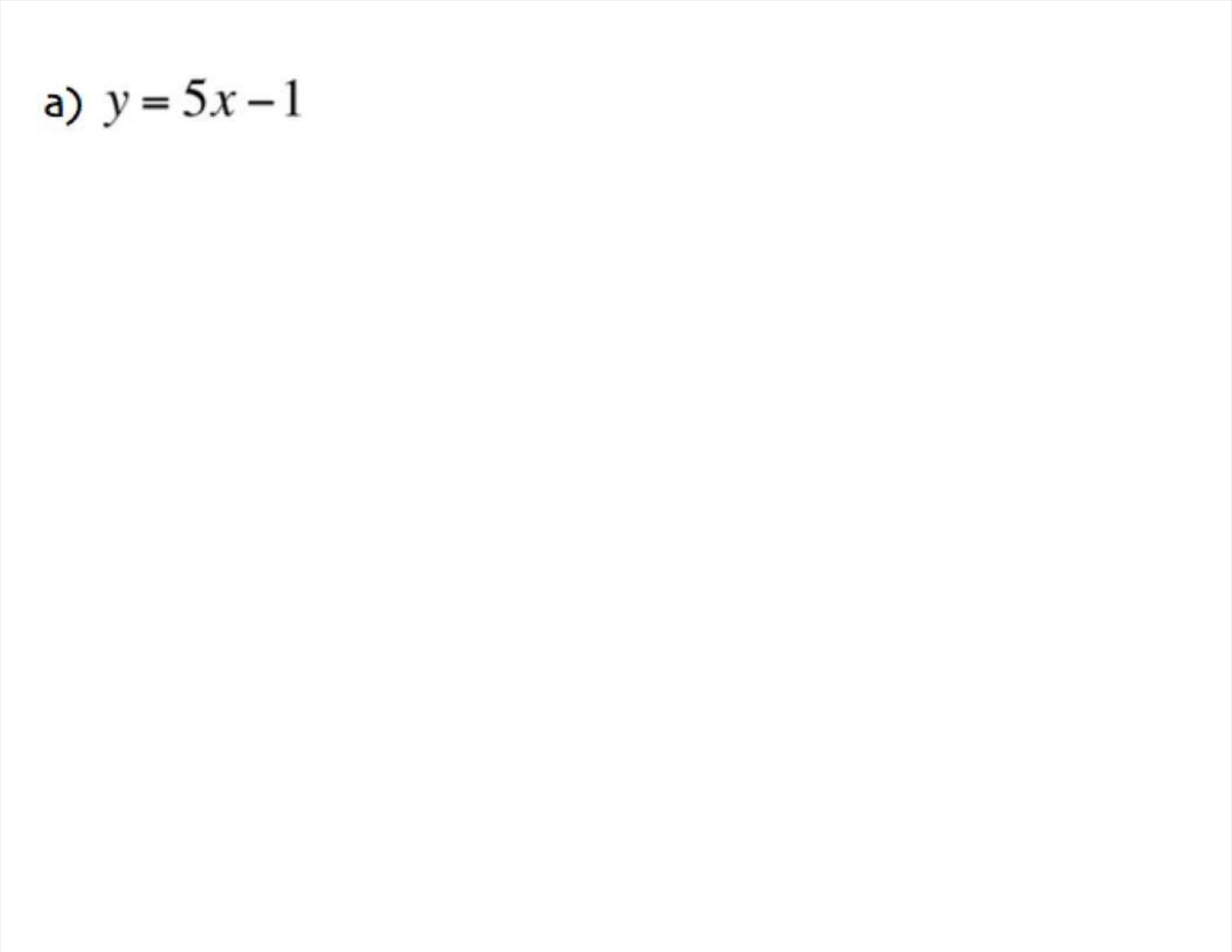
b) $f\left(x\right) = x\log_5 x$

c) $g\left(x\right) = \cos^2 x$

d) $f\left(x\right) = \dfrac{e^{7x}}{3^x}$

e) $y = \ln\left(\sec 2x\right)$

f) $y = e^5 - 5e^4 + 7$



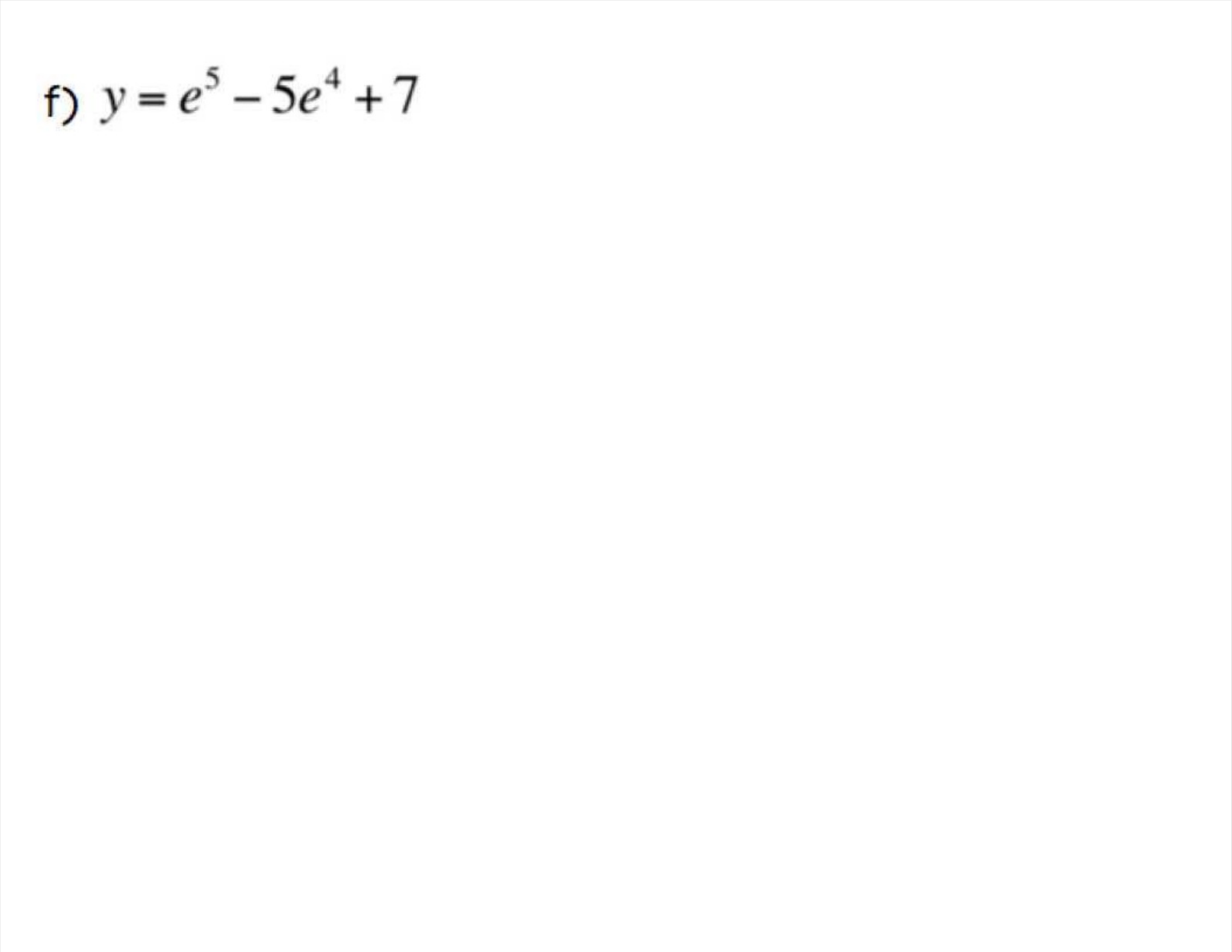
a) $y = 5x - 1$

b) $f\left(x\right) = x\log_5 x$

c) $g(x)=\cos^2 x$

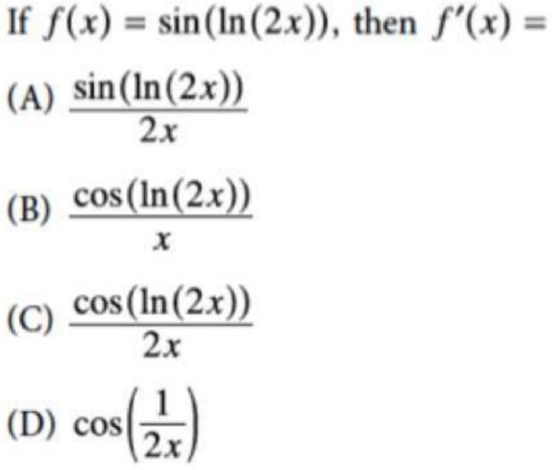
d) $f\left(x\right)=\frac{e^{7x}}{3^{x}}$

e) $y = \ln\left(\sec 2x\right)$



f) $y = e^5 - 5e^4 + 7$

ex:



If $f(x) = \sin(\ln(2x))$, then $f'(x) =$

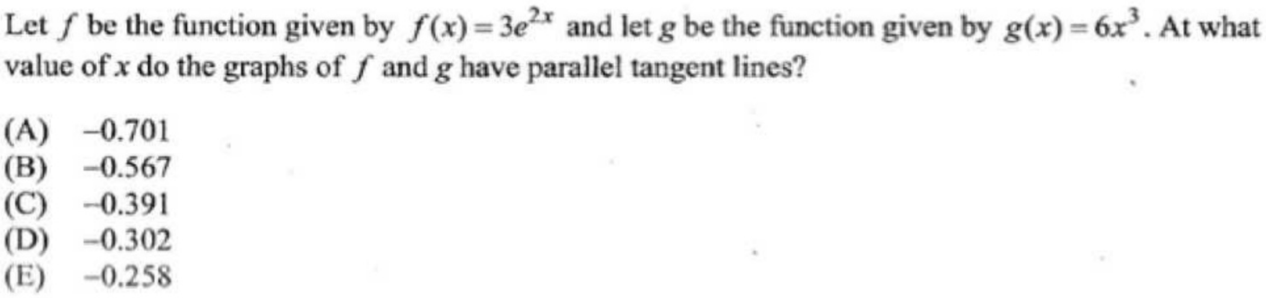
(A) $\dfrac{\sin(\ln(2x))}{2x}$

(B) $\dfrac{\cos(\ln(2x))}{x}$

(C) $\dfrac{\cos(\ln(2x))}{2x}$

(D) $\cos\left(\dfrac{1}{2x}\right)$

ex:



Let f be the function given by $f(x)=3e^{2x}$ and let g be the function given by $g(x)=6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.701
(B) -0.567
(C) -0.391
(D) -0.302
(E) -0.258

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Let f be the function given by $f(x) = \ln \dfrac{x}{x-1}$.

(a) What is the domain of f ?

(b) Find the value of the derivative of f at $x = -1$.