2.4 Chain Rule Cont.

ex: Differentiate.

a) $y = \csc(7x)$

b) $y = \dfrac{6}{(3x^2 - 5)^5}$

THEOREM 2.13 Derivative of the Natural Logarithmic Function

Let u be a differentiable function of x.

1. $\frac{d}{dx}[\ln x] =$

2. $\frac{d}{dx}[\ln u] =$

ex: Differentiate.

a) $y = \ln(2x)$

ex: Differentiate.

b) $y = x \ln x$

ex: Differentiate.

c) $y = \ln\sqrt{x^2 + 8e^x}$

ex: Differentiate.

d) $f(x) = \ln(x \sin x)$

THEOREM 2.14 Derivative Involving Absolute Value

If u is a differentiable function of x such that $u \neq 0$, then

$$\frac{d}{dx}[\ln|u|] =$$

ex: Differentiate.

a) $f(x) = \ln\left|\csc x\right|$

ex: Differentiate.

b) $f\left(x\right)=\ln\left|\frac{\cos x-2}{e^{x}}\right|$

ex: Differentiate.

c) $y = |\ln x|$

Definition of Logarithmic Function to Base *a*

If a is a positive real number ($a \neq 1$) and x is any positive real number, then the **logarithmic function to the base *a*** is denoted by $\log_a x$ and is defined as

$$\log_a x =$$

ex: Differentiate.

a) $y = \log_7 x$

ex: Differentiate.

b) $y = \log_6 \dfrac{x\sqrt{x-2}}{5}$

Derivative of the Exponential Function

Let u be a differentiable function of x.

$$\frac{d}{dx}\left[a^x\right] =$$

$$\frac{d}{dx}\left[e^x\right] =$$

$$\frac{d}{dx}\left[a^u\right] =$$

$$\frac{d}{dx}\left[e^u\right] =$$

ex: Differentiate.

a) $y = 3^x$

b) $y = e^{5x}$

ex: Differentiate.

c) $y = 5^{\ln x}$

d) $f\left(x\right) = 4^{\log_4 x}$

ex: Differentiate.

e) $y = \dfrac{3}{e^x}$

f) $y = \ln\left(\dfrac{1+e^x}{1-e^x}\right)$

ex: Differentiate.

g) $y = e^x \ln x$

h) $f(x) = \ln e^{777x}$

ex: Write the equation of the tangent line to

$y = e^{4x}$ at $x = \ln 2$.

ex: Write an equation of the tangent line to $y = \frac{\ln x}{4x}$ at x=1.

ex: Write an equation of the normal line to $y = \frac{\ln x}{4x}$ at x=1.

Summary of Differentiation Rules

General Differentiation Rules

Let u and v be differentiable functions of x.

Constant Rule:

$\frac{d}{dx}[c] = 0$, c is a real number.

(Simple)Power Rule:

$\frac{d}{dx}[x^n] = nx^{n-1}, \frac{d}{dx}[x] = 1$, n is a rational number.

Constant Multiple Rule:

$\frac{d}{dx}[cu] = cu'$, c is a real number.

Sum or Difference Rule:

$\frac{d}{dx}[u \pm v] = u' \pm v'$

Product Rule:

$\frac{d}{dx}[uv] = uv' + vu'$

Quotient Rule:

$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$

Chain Rule:

$\frac{d}{dx}[f(u)] = f'(u)u'$

General Power Rule:

$\frac{d}{dx}[u^n] = nu^{n-1}u'$, n is a rational number.

Derivatives of Trigonometric Functions

$\frac{d}{dx}[\sin x] = \cos x$

$\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$

$\frac{d}{dx}[\cos x] = -\sin x$

$\frac{d}{dx}[\cot x] = -\csc^2 x$

$\frac{d}{dx}[\csc x] = -\csc x \cot x$

Derivatives of Exponential and Logarithmic Functions

$\frac{d}{dx}[e^x] = e^x$

$\frac{d}{dx}[\ln x] = \frac{1}{x}, \quad x > 0$

$\frac{d}{dx}[a^x] = (\ln a)a^x$,

a is a positive real number ($a \neq 1$).

$\frac{d}{dx}[\log_a x] = \frac{1}{(\ln a)x}$,

a is a positive real number ($a \neq 1$).

ex: Find the derivative.

a) $y = 5x - 1$

d) $f(x) = \frac{e^{7x}}{3^x}$

b) $f(x) = x \log_5 x$

e) $y = \ln(\sec 2x)$

c) $g(x) = \cos^2 x$

f) $y = e^5 - 5e^4 + 7$

a) $y = 5x - 1$

b) $f\left(x\right)=x\log_5 x$

c) $g(x) = \cos^2 x$

d) $f\left(x\right)=\dfrac{e^{7x}}{3^{x}}$

e) $y = \ln\left(\sec 2x\right)$

f) $y = e^5 - 5e^4 + 7$

ex:

If $f(x) = \sin(\ln(2x))$, then $f'(x) =$

(A) $\dfrac{\sin(\ln(2x))}{2x}$

(B) $\dfrac{\cos(\ln(2x))}{x}$

(C) $\dfrac{\cos(\ln(2x))}{2x}$

(D) $\cos\left(\dfrac{1}{2x}\right)$

ex:

Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) −0.701
(B) −0.567
(C) −0.391
(D) −0.302
(E) −0.258

FR 12

Let f be the function given by $f(x) = \ln \dfrac{x}{x-1}$.

(a) What is the domain of f ?

(b) Find the value of the derivative of f at $x = -1$.