110

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$$

$$8a = 4 + b$$

$$\lim_{x \to 2^-} (3ax^2) = \lim_{x \to 2^+} (2x) = f'(2)$$

$$12a = 4$$

$$a = \frac{1}{3}$$

There is only one more quiz and one more
test for the quarter grade !! Make them count!!

Quiz average: 89%

Normal line = perpendicular line

slope will be the negative reciprocal

2.) $\overset{q}{2}x^3 - 3x^2 - 12x + \overset{p}{20} = 0$

2	2	-3	-12	20
	↓	4	2	-20
	2	1	-10	0

$2x^2 + x - 10 = 0$

$(2x+5)(x-2) = 0$

$(x-2)$

$$\frac{\pm 1, 2, 4, 5, 10, 20}{\pm 1, \pm 2}$$

2c)

$x = 2, -1$

$(-1, 27)$

$2(-1)^3 - 3(-1)^2 - 12(-1) + 20$

$-2 + -3 + 12 + 20$

3a. $\lim_{x \to 2^-} 2x+1 = \lim_{x \to 2^+} \left(\frac{1}{2}x^2 + k\right) = f(2)$

$5 = 2 + k$

$3 = k$

$$f(x) = \begin{cases} 2x+1, & x \leq 2 \\ \frac{1}{2}x^2+3, & x > 2 \end{cases}$$

$$\lim_{x \to 2^-} 2 = \lim_{x \to 2^+} x = f'(2)$$

$$2 = 2 \checkmark$$

$$f(x) = \begin{cases} 2x+1, & x \le 2 \\ \frac{1}{2}x^2+4, & x > 2 \end{cases} \quad \begin{matrix} 5 \\ 6 \end{matrix}$$

at $x=2$ $f(x)$ is not cont. $\therefore$ $f(x)$ is not diff.

2.3 Product & Quotient Rules

ex: Find the derivative.

a) $y=(x+1)(x^2-3)$

$y = x^3 + x^2 - 3x - 3$

$y' = 3x^2 + 2x - 3$

b) $g(x)=\dfrac{2x^2-1}{x} = \dfrac{2x^2}{x} - \dfrac{1}{x}$

$g(x) = 2x - x^{-1}$

$g'(x) = 2 + 1x^{-2}$

a.) $y = \overset{f}{(x+1)}\overset{g}{(x^2-3)}$

$y' = (x+1)(2x) + (1)(x^2-3)$

$y' = 2x^2 + 2x + x^2 - 3$

c) $y = e^x \sin x$

d) $f(x) = \dfrac{x^2 + 1}{2x - 1}$

THEOREM 2.8 The Product Rule

The product of two differentiable functions f and g is itself differentiable. Moreover, the derivative of fg is

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$$

sum of 2 products

ex: Which function(s) are good candidates for the product rule?

- ✗ $f(x)=4\sin x$
- ✓ $y=xe^{x}$
- tie $y=(x+1)(x^2-3)$
- ✗ $y=2x^3$ $\quad y' = 2\cdot 3x^2 + \cancel{x^3\cdot 0}$
- ✗ $g(x)=\sin 2x$

ex: Find the derivative.

a) $y = xe^x$ $\quad f \cdot g$

$y' = x \cdot e^x + e^x \cdot 1$

or

$y' = e^x(x+1)$

$\cos^2 x - \sin^2 x$

$\cos 2x$

b) $g(x) = \sin 2x = (2\sin x)(\cos x)$

$g'(x) = 2\sin x(-\sin x) + \cos x \cdot 2\cos x$

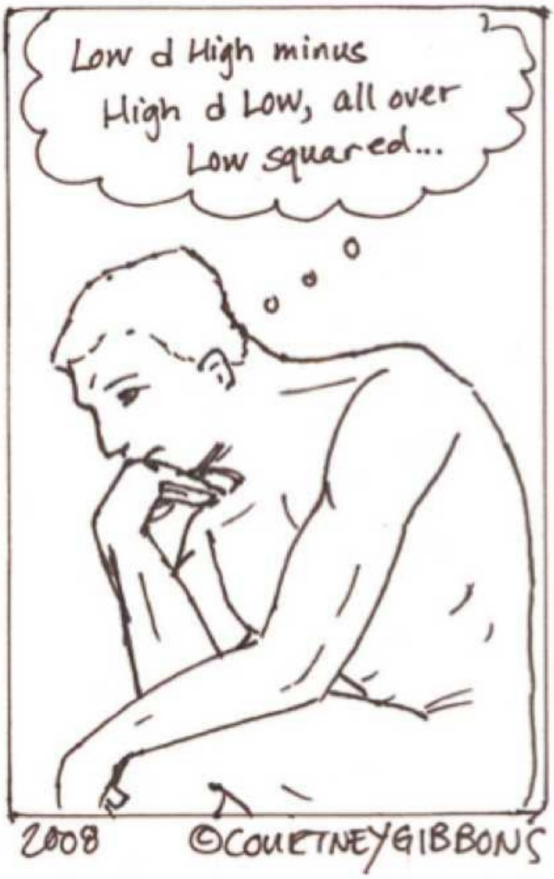
$= -2\sin^2 x + 2\cos^2 x$

THEOREM 2.9 The Quotient Rule

The quotient f/g of two differentiable functions f and g is itself differentiable at all values of x for which $g(x) \neq 0$.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

Remembering The Quotient Rule...



$$\frac{Hi}{Lo}$$

$$\frac{Lo \cdot dHi - Hi \cdot dLo}{(Lo)^2}$$

ex: Which function(s) are good candidates for the quotient rule?

- $g(x)=\frac{5}{x}$ $= 5x^{-1}$
- $y=\frac{x}{5}$ $= \frac{1}{5}x$
- $y=\frac{e^x}{x+1}$
- $f(x)=\frac{x+1}{x}$ $= 1+x^{-1}$
- $y=\tan x$

ex: Find the derivative.

a) $y = \dfrac{e^x}{x+1}$

$$y' = \frac{(x+1)e^x - e^x \cdot 1}{(x+1)^2}$$

$$y' = \frac{xe^x + e^x - e^x}{(x+1)^2} = \frac{xe^x}{(x+1)^2}$$

ex: Find the derivative.

b) $f(x) = x^3\left(1 - \frac{2}{x+1}\right)$

ex: Find the derivative.

c) $f(x)=\dfrac{3-\dfrac{1}{x+5}}{x-1}$

ex: Find the derivative.

c) $y = \tan x$

$$y = \frac{\sin x}{\cos x}$$

$$y' = \frac{\cos x \cos x + \sin x(-\sin x)}{(\cos x)^2}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

ex: Find the derivative.

d) $y = \cot x$

$y' = -\csc^2 x$

ex: Find the derivative.

e) $y = \sec x$

$$y = \frac{1}{\cos x}$$

$$y' = \frac{\cos x \cdot 0 - (1)(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \tan x \sec x$$

ex: Find the derivative.

f) $y = \csc x$

$y' = -\cot x \csc x$

Trigonometric Derivatives

$$\frac{d}{dx}[\sin x] =$$

$$\frac{d}{dx}[\csc x] =$$

$$\frac{d}{dx}[\cos x] =$$

$$\frac{d}{dx}[\sec x] =$$

$$\frac{d}{dx}[\tan x] =$$

$$\frac{d}{dx}[\cot x] =$$

Remembering The Derivatives of Tangent, Cotangent, Secant and Cosecant...

*MEMORIZE THIS CHART

$\tan x$	$\sec x$	$\sec x$
$\cot x$	$\csc x$	$-\csc x$

ex: Find the equation of the tangent line to $y = \cancel{x^2 \sec x}$ $x\tan x$

at $x = \dfrac{\pi}{4}$.

$y' = x\sec^2 x + \tan x \cdot 1$

$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{4}\left(\sec\frac{\pi}{4}\right)^2 + \tan\frac{\pi}{4}$

$= \frac{\pi}{2} + 1 = \frac{\pi+2}{2}$

$\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$

$y - y_1 = \underline{m}(x - x_1)$

$y - \frac{\pi}{4} = \left(1 + \frac{\pi}{2}\right)\left(x - \frac{\pi}{4}\right)$

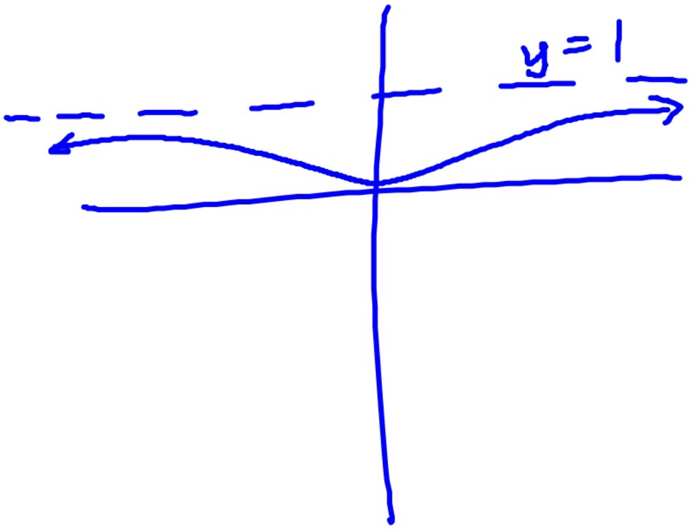
ex: Find the point(s), if any, at which $f(x) = \dfrac{x^2}{x^2+1}$ has a horizontal tangent.

$$f'(x) = \frac{2x}{(x^2+1)^2}$$

$$0 = \frac{2x}{(x^2+1)^2}$$

$$0 = x$$

$$(0,0)$$



ex:

$H(x) = ef(x) + \pi x$

$H'(x) = ef'(x) + \pi$

$H'(0) = ef'(0) + \pi$

$= -e + \pi$

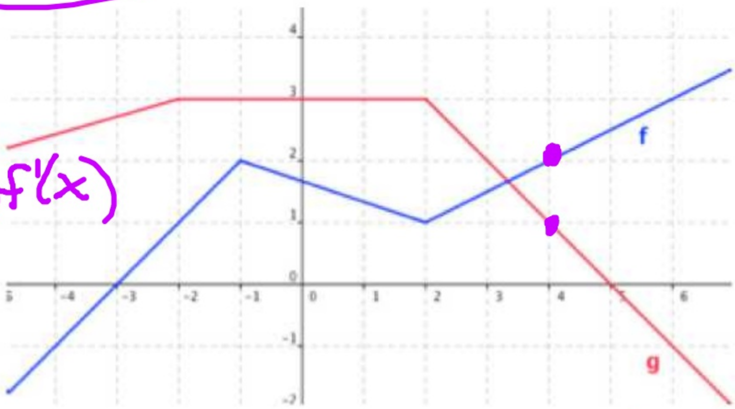
x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	1	–1	2	5
1	–1	2	4	0
2	7	3	11	0.5

Based on the values in the table above, if $H(x) = ef(x) + \pi x$, then $H'(0) =$

(A) $\pi - e$ (B) $e^x + \pi x$ (C) $e + \pi$ (D) e (E) $e^{-1} + \pi$

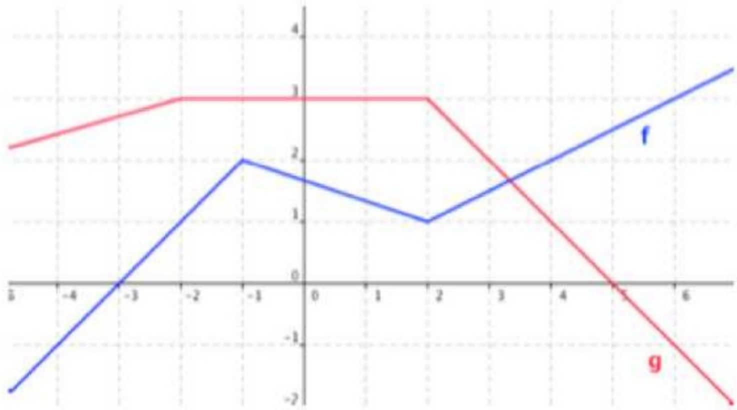
ex: Let $p(x) = f(x)g(x)$ and $q(x) = \dfrac{f(x)}{g(x)}$.

$p'(x) =$
$f(x)g'(x) + g(x)f'(x)$

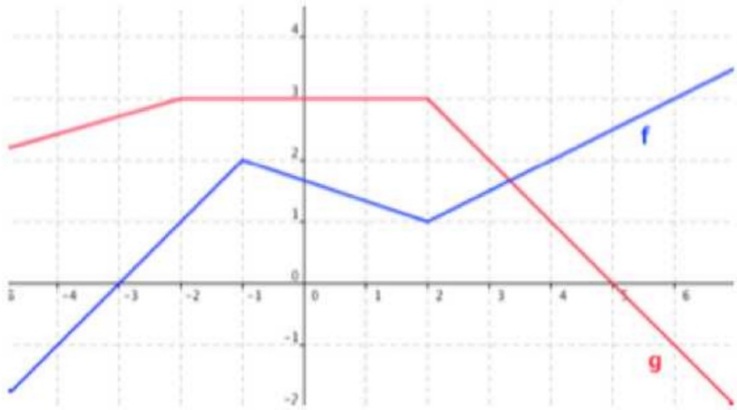


1. $p'(4) = f(4)g'(4) + g(4)f'(4)$
$(2)\cdot(-1) + (1)\left(\frac{1}{2}\right) = -\frac{3}{2}$

*See printout.

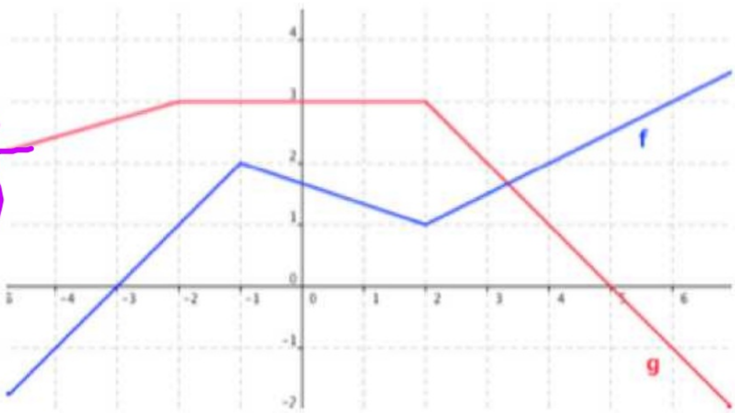


2. $p'(4)$

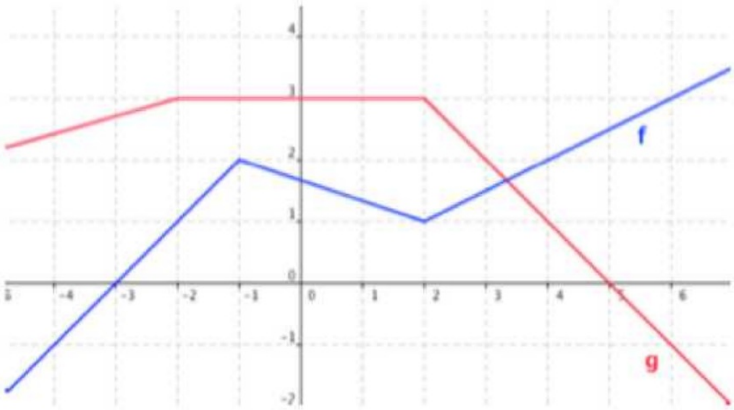


3. $p'(-1)$

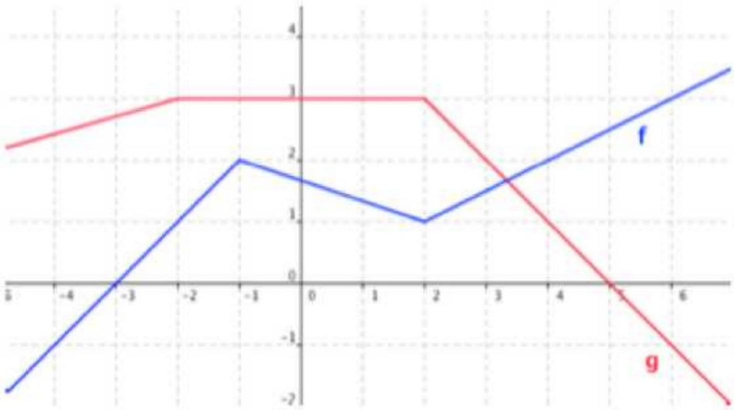
$$q(x) = \frac{f(x)}{g(x)}$$



4. $q'(3)$



5. $q'(-2)$



6. $q'(6)$

Higher Order Derivatives

First derivative:	y',	$f'(x)$,	$\frac{dy}{dx}$,	$\frac{d}{dx}[f(x)]$,	$D_x[y]$
Second derivative:	y'',	$f''(x)$,	$\frac{d^2y}{dx^2}$,	$\frac{d^2}{dx^2}[f(x)]$,	$D_x^2[y]$
Third derivative:	y''',	$f'''(x)$,	$\frac{d^3y}{dx^3}$,	$\frac{d^3}{dx^3}[f(x)]$,	$D_x^3[y]$
Fourth derivative:	$y^{(4)}$,	$f^{(4)}(x)$,	$\frac{d^4y}{dx^4}$,	$\frac{d^4}{dx^4}[f(x)]$,	$D_x^4[y]$
$\vdots$					
nth derivative:	$y^{(n)}$,	$f^{(n)}(x)$,	$\frac{d^ny}{dx^n}$,	$\frac{d^n}{dx^n}[f(x)]$,	$D_x^n[y]$

ex: Find the indicated derivative.

a) $f(x) = 3x^4 + 2, \qquad f'''(x) = ?$

$f'(x) = 12x^3$

$f''(x) = 36x^2$

$f'''(x) = 72x$

ex: Find the indicated derivative.

b) $y = \sin x, \qquad \dfrac{d^2 x}{dx^2} = ?$

ex: Find the indicated derivative.

c) $y = \sin x$, $\frac{d^5x}{dx^5} = ?$

$y' = \cos x$

$y'' = -\sin x$

$y''' = -\cos x$

$y^{(4)} = \sin x$

$y^{(5)} = \cos x$

$$\begin{array}{r|l} & 1 \\ 4 & 5 \\ & 4 \\ \hline & r.1 \end{array}$$

ex: Find the indicated derivative.

d) $y = \sin x,\qquad \dfrac{d^{205}x}{dx^{205}} = ?$

ex: Find the indicated derivative.

e) $g(x) = \cos x, \qquad g^{(163)}(x) = ?$

ex: Find the indicated derivative.

f) $y = e^{x}$, $\qquad y^{(1111)} = ?$

FR 5

Let f be the function given by $f(x)=\frac{2x-5}{x^2-4}$.

a. Find the domain of f.
b. Write an equation for each vertical and each horizontal asymptote for the graph of f.
c. Find $f'(x)$.
d. Write an equation for the line tangent to the graph of f at the point (0,f(0)).

FR 18

Let f be the function that is given by $f(x) = \dfrac{ax+b}{x^2-c}$ and that has the following properties.

(i) The graph of f is symmetric with respect to the y-axis.

(ii) $\lim_{x \to 2^+} f(x) = +\infty$

(iii) $f'(1) = -2$

(a) Determine the values of a , b , and c .

(b) Write an equation for each vertical and each horizontal asymptote of the graph of f.

(c) Sketch the graph of f in the xy-plane provided below.