46. $f(x) = x^3$; $c = -2$

88. B.

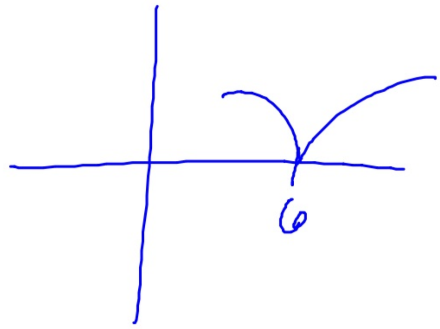
47.) $\lim_{x \to 6} \frac{-x^2 + 36}{x - 6}$

$c = 6$

$f(x) = -x^2$

67. $f(x) = (x-6)^{2/3}$ $c = 6$

$$f'(6) = \lim_{x \to 6} \frac{(x-6)^{2/3} - 0}{(x-6)^1}$$



$$= \lim_{x \to 6} \frac{1}{(x-6)^{1/3}}$$

$$\lim_{x \to 6^-} \frac{1}{(x-6)^{1/3}} \neq \lim_{x \to 6^+} \frac{1}{(x-6)^{1/3}}$$

dne.
not diff.
at $x = 6$

2.2 Basic Differentiation Rules

*AKA "Short Cut Rules"

THEOREM 2.2 The Constant Rule

The derivative of a constant function is 0. That is, if c is a real number, then

$$\frac{d}{dx}[c] = 0$$

ex: Differentiate.

$$y = 234897\pi \quad y' = 0$$

THEOREM 2.3 The Power Rule

If n is a rational number, then the function $f(x) = x^n$ is differentiable and

$$\frac{d}{dx}[x^n] = n \cdot x^{n-1}$$

ex: Differentiate.

a) $f(x) = x^3$

$f'(x) = 3x^2$

b) $f(x) = x^{20}$ $f'(x) = 20x^{19}$

ex: Differentiate.

c) $g(x) = \frac{1}{x} = x^{-1}$

$g'(x) = -1 \cdot x^{-2} = \frac{-1}{x^2}$

d) $h(x) = \sqrt{x} = x^{1/2}$

$h'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

e) $y = \frac{1}{\sqrt{x}} = x^{-1/2}$

$y' = -\frac{1}{2} x^{-3/2}$

THEOREM 2.4 The Constant Multiple Rule

If f is a differentiable function and c is a real number, then cf is also differentiable and

$$\frac{d}{dx}[cf(x)] = c \cdot f'(x)$$

ex: Differentiate.

a) $y = 30x^7$

$y' = 30 \cdot 7x^6 = 210x^6$

$y = \frac{3}{2x^2} = \frac{3}{2}x^{-2}$

$y' = -3x^{-3}$

b) $g(x) = \frac{4}{\sqrt[3]{x}} = 4x^{-1/3}$ $g'(x) = -\frac{4}{3}x^{-4/3}$

$$\lim_{h\to 0} \frac{(2+h)^5 - 32}{h}$$

$f(x) = x^5$ $\quad f'(x) = 5x^4$

$c = 2$ $\quad f'(2) = 80$

ex: Differentiate.

c) $g(x) = \pi$

d) $m(x) = 4x$

THEOREM 2.5 The Sum and Difference Rules

The sum (or difference) of two differentiable functions f and g is itself differentiable. Moreover, the derivative of $f + g$ (or $f - g$) is the sum (or difference) of the derivatives of f and g.

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

ex: Differentiate.

a) $y = 3x^4 - 2x + \pi$ $\quad y' = 12x^3 - 2$

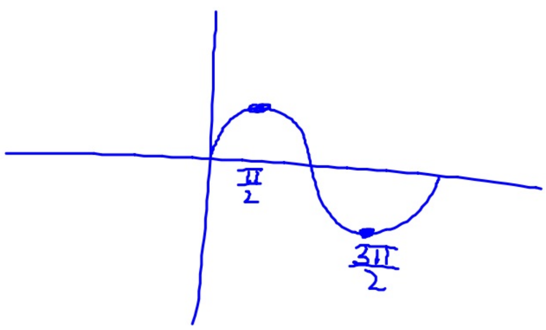
ex: Differentiate.

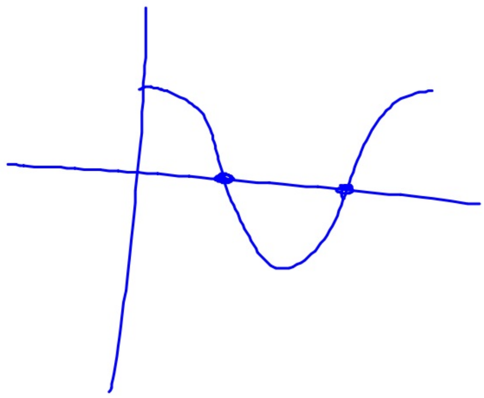
b) $f(x) = \pi^2 + \frac{1}{\pi} + \sqrt{\pi}$

c) $s(x) =$

THEOREM 2.6 Derivatives of Sine and Cosine Functions

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$





$$\lim_{h \to 0} \frac{\cos\left(\frac{3\pi}{2}+h\right) - \cos\frac{3\pi}{2}}{h} = m$$

$f(x) = \cos x \qquad f'(x) = -\sin x$

$c = \frac{3\pi}{2} \qquad f'\left(\frac{3\pi}{2}\right) = 1$

ex: Differentiate.

a) $y = 4\cos x - 2\sin x + 3$

$y' = -4\sin x - 2\cos x$

THEOREM 2.7 **Derivative of the Natural Exponential Function**

$$\frac{d}{dx}[e^x] = e^x$$

ex: Differentiate.

a) $y = 3e^x$ $\qquad y' = 3e^x$

ex: Differentiate.

b) $y = x^2 - e^x$

$y' = 2x - e^x$

c) $f(x) = \cos x + 5e^x$

$f'(x) = -\sin x + 5e^x$

ex: Differentiate.

d) $y = x\left(x^2 + 5\right)$

$y = x^3 + 5x^1$

$y' = 3x^2 + 5$

e.) $f(x) = \dfrac{x^2 + 5}{x}$

$= \dfrac{x^2}{x} + \dfrac{5}{x}$

$f(x) = x + 5x^{-1}$

$f'(x) = 1 - 5x^{-2}$

ex: Differentiate.

d) $f\left(x\right)=\dfrac{x^2+5}{x}$

ex: Find the slope at the given point.

a) $f(x) = -5x^4 - 2x^3 + 3\pi, \quad x = -1$

$f'(x) = -20x^3 - 6x^2$; $f'(-1) = 14$

$f'(-1) = -20(-1)^3 - 6(-1)^2$

$= 20 - 6$

$f'(-1) = 14$

ex: Find the slope at the given point.

b) $g(x) = -e^x, \quad x = 0$

$g(x) = -1e^x$

$g'(x) = -e^x$

$g'(0) = -1$

ex: Write an equation of the tangent line at the given point.

a) $y = \cos x, \quad x = \frac{3\pi}{4}$

$\left(\frac{3\pi}{4}, -\frac{\sqrt{2}}{2}\right)$

point

$y' = -\sin x$

$y'\left(\frac{3\pi}{4}\right) = -\sin\frac{3\pi}{4} = -\frac{\sqrt{2}}{2} = m$

slope

$y + \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}\left(x - \frac{3\pi}{4}\right)$

ex: Write an equation of the tangent line at the given point.

b) $f(x) = 3 - \dfrac{3}{5x}$ $\quad x = \dfrac{3}{5}$

$\left(\dfrac{3}{5}, 2\right)$

$f(x) = 3 - \frac{3}{5}x^{-1}$

$f'(x) = 0 + \frac{3}{5}x^{-2}$

$f'(x) = \dfrac{3}{5x^2}$

$f'\left(\frac{3}{5}\right) = \dfrac{3}{5\left(\frac{3}{5}\right)^2} = \dfrac{3}{\cancel{5}\cdot\frac{9}{\cancel{25}\,5}} = 3\cdot\frac{5}{9} = \frac{5}{3}$ m

$y - 2 = \frac{5}{3}\left(x - \frac{3}{5}\right)$

ex: Find all points, if any, at which f(x) has a horizontal tangent line.

slope = 0

a) $f(x) = \sin x, \quad [0, 2\pi)$

$f'(x) = \cos x$

$0 = \cos x$

$\frac{\pi}{2}, \frac{3\pi}{2} = x$

ex: Find all points, if any, at which f(x) has a horizontal tangent line.

b) $y = e^x - 2$

ex: Find an equation of a line that is tangent to $f(x) = 5x^2 + 3$ and parallel to $5x - y = 4$.

$m = 5$

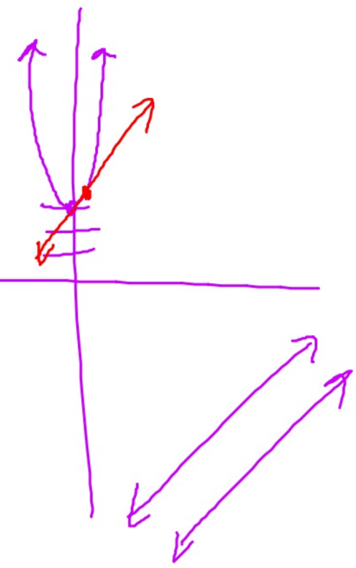
$f'(x) = 10x$

$5 = 10x$

$\frac{1}{2} = x$

$\left(\frac{1}{2}, \frac{17}{4}\right)$

$y - \frac{17}{4} = 5\left(x - \frac{1}{2}\right)$



ex: Find the value of k such that the line $\cancel{y = x + 4}$ $y = 6x + 1$

is tangent to $\cancel{f(x) = k\sqrt{x}}$.

$f(x) = x^2 + K$

$x^2 + K = 6x + 1$

$2x = 6$

$x = 3$

$K = 10$