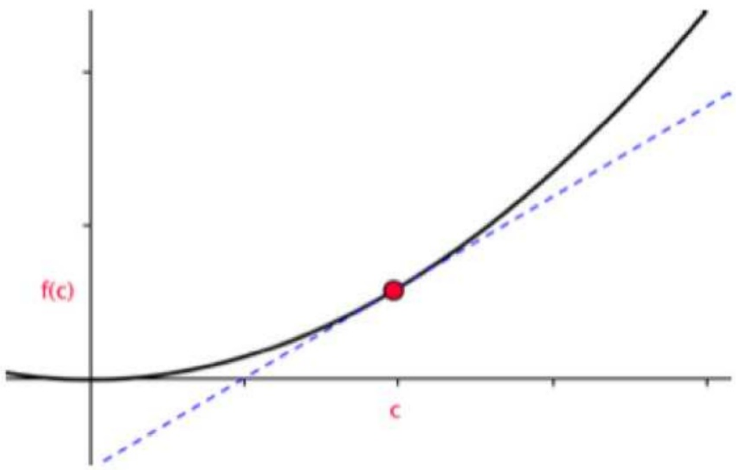
2.1 Definition Of A Derivative

The Tangent Line Problem

Task: Write the equation of the tangent line to f(x) at x=c.



DEFINITION OF TANGENT LINE WITH SLOPE m

If f is defined on an open interval containing c, and if the limit

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = m$$

exists, then the line passing through $(c, f(c))$ with slope m is the **tangent line** to the graph of f at the point $(c, f(c))$.

*The slope of the tangent line to the graph of f at the point (c, f(c)) with slope m is also known as the <u>slope of the graph of f at x=c.</u>

ex: Find the slope of the tangent line at the given point.

a) $f(x) = x^2 + 1, \quad (2,5)$

ex: Find the slope of the tangent line at the given point.

c) $f(x) = x^2 + 1, \quad (0,1)$

ex: Find the slope of the tangent line at the given point.

d) $f(x) = 3, \quad (21,3)$

ex: Interpret the expression.

$$\lim_{h \to 0} \frac{f(6+h) - f(6)}{h} = 100$$

Derivative - a formula used to find the slope of a tangent line

Vocab:

- differentiation - the process of finding a derivative
- differentiate - to find a derivative
- differntiable - a derivative exists

$$f'(x), \quad \frac{dy}{dx}, \quad y', \quad \frac{d}{dx}[f(x)], \quad D_x[y].$$

Notation for derivatives

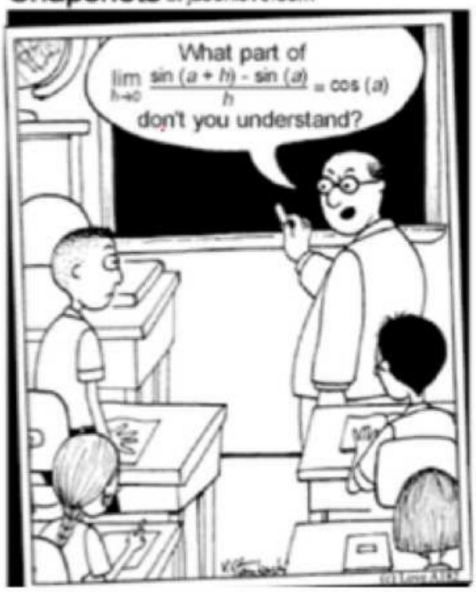
DEFINITION OF THE DERIVATIVE OF A FUNCTION

The **derivative** of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

provided the limit exists. For all x for which this limit exists, f' is a function of x.

Snapshots at jasonlove.com



Derivative Synonyms:

- rate of change
- slope of a tangent line

ex: Find the derivative using the limit process.

a) $f(x) = x^2$

ex: Find the derivative using the limit process.

b) $f(x) = \sqrt{x}$

ex: Find the derivative using the limit process.

c) $f(x) = \dfrac{1}{x}$

ex: Find the slope at the given point.

a) $f(x) = \frac{1}{x}, \quad \left(10, \frac{1}{10}\right)$

ex: Find the slope at the given point.

b) $f(x) = \sqrt{x}, \quad (0,0)$

ex: Interpret the expression.

$$g'(22) = 5$$

- Alternate Form Of The Derivative

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

ex: Use the alternate form to find the slope of the curve at the given x-value.

a) $f(x) = x^2 + 1, \quad (2,5)$

ex: Use the alternate form to find the slope of the curve at the given x-value.

b) $f(x)=17-4x, \quad (1,13)$

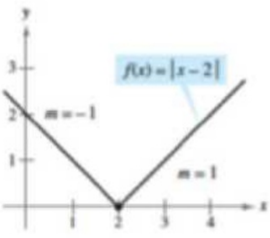
ex: Use the alternate form to find the slope of the curve at the given x-value.

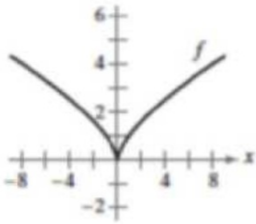
c) $f(x)=|x|, \quad (0,0)$

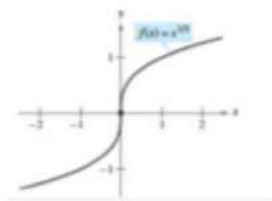
ex: Use the alternate form to find the slope of the curve at the given x-value.

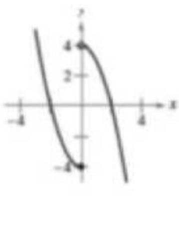
e) $f(x) = \sqrt{x}, \quad (0,0)$

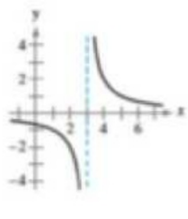
A function f is NOT DIFFERENTIABLE at...











If f is differentiable at x=c, then f must be continuous at x=c.

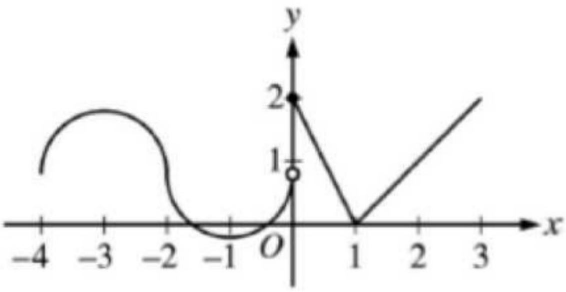
BUT...

If f is continuous at x=c, f may or MAY NOT be differentiable at x=c.

MORAL OF THE STORY:

DIFFERNTIABILITY IMPLIES CONTINUITY

ex:



Graph of f

The graph of the piecewise-defined function f is shown in the figure above. The graph has a vertical tangent line at $x = -2$ and horizontal tangent lines at $x = -3$ and $x = -1$. What are all values of x, $-4 < x < 3$, at which f is continuous but not differentiable?

(A) $x = 1$

(B) $x = -2$ and $x = 0$

(C) $x = -2$ and $x = 1$

(D) $x = 0$ and $x = 1$

ex: The limit below represents the derivative of f at x=c for a function f and a number c. Find f and c.

a) $$\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{2}+h\right)-1}{h}$$

b) $$\lim_{x\to e}\frac{\ln x-1}{x-e}$$

ex:

What is $\lim_{h \to 0} \frac{\cos\left(\frac{3\pi}{2} + h\right) - \cos\left(\frac{3\pi}{2}\right)}{h}$?

(A) 1

(B) $\frac{\sqrt{2}}{2}$

(C) 0

(D) -1

(E) The limit does not exist.