Revised chapter 1 syllabus is posted on my website AND by Garfield

next quiz: Wednesday

square root

D: $[0,\infty)$

reciprocal of square

R: $(0,\infty)$

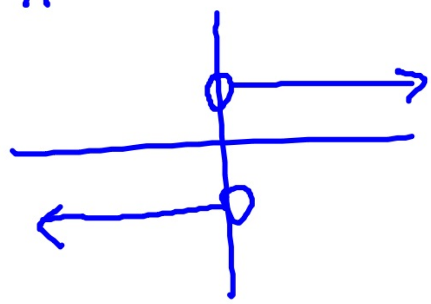
128. a.) $1/2$

b.) 1

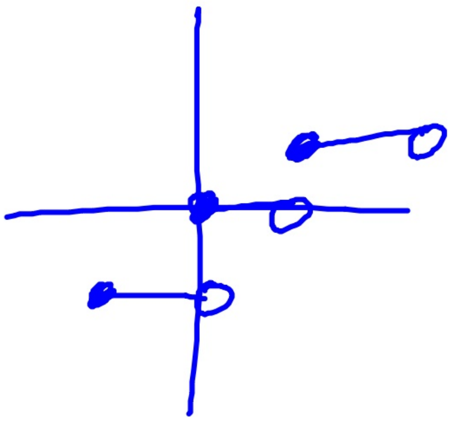
c.) dne

d.) 3

13.) $$\lim_{x \to 0^-} \frac{|x|}{x} = -1$$



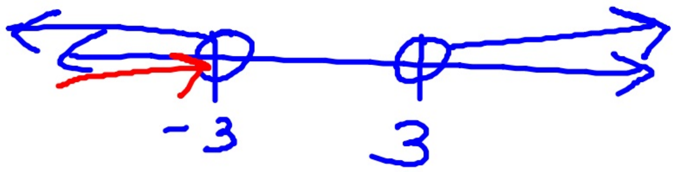
23.) $\lim_{x \to 3} \left(2 - [-x]\right)$



11.) $\lim\limits_{x \to -3^-} \frac{x}{\sqrt{x^2-9}}$

dne $(-\infty)$

$$\frac{-3.01}{\sqrt{(-3.01)^2-9}}$$



$$\frac{-3.01}{\text{small}} \qquad \frac{-3}{\frac{1}{10000}}$$

15.) $$\lim_{h \to 0^-} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$\lim_{h \to 0^-} \frac{x-(x+h)}{x(x+h)} \cdot \frac{1}{h}$$

$$\lim_{h \to 0^-} \frac{-1}{x(x+h)}$$

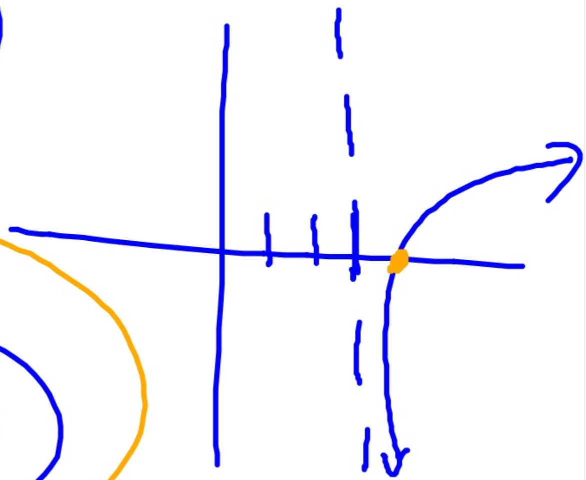
$$\frac{-1}{x^2}$$

$$\lim_{x \to 3^+} \ln(x-3)$$

$-\infty$

$\ln(3.01-3)$

$\ln(.01)$



1.5/1.6 Infinite Limits and Limits at Infinity

REVIEW:

Finding Horizontal Asymptotes - if f(x) is a rational function...

$$f(x) = \frac{ax^n + \dots}{bx^m + \dots}$$

$\leftarrow$ nth degree polynomial

$\leftarrow$ mth degree polynomial

$$\frac{x^3}{e^x}$$

Bobo

Eats dc

Botn

1. If $n < m$, then the x-axis is the horizontal asymptote.
2. If $n = m$, then the horizontal asymptote is the line $y = \frac{a}{b}$
3. If $n > m$, then there is no horizontal asymptote.

REMEMBER THE ACRONYM: BOBO BOTN EATSDC

*If f(x) is not a rational function but comes in the form of a fraction compare the magnitudes of the numerator and denominator and use "BOBO."

Finding Vertical Asymptotes - Vertical asymptotes are vertical lines which correspond to the zeroes of the denominator of a simplified rational function. (They can also arise in other types of functions.)

*WATCH OUT FOR HOLES!

ex: State the horizontal and vertical asymptotes.

a) $f(x)=\frac{x-1}{x^2+7x-8}$ $= \frac{1}{x+8}$

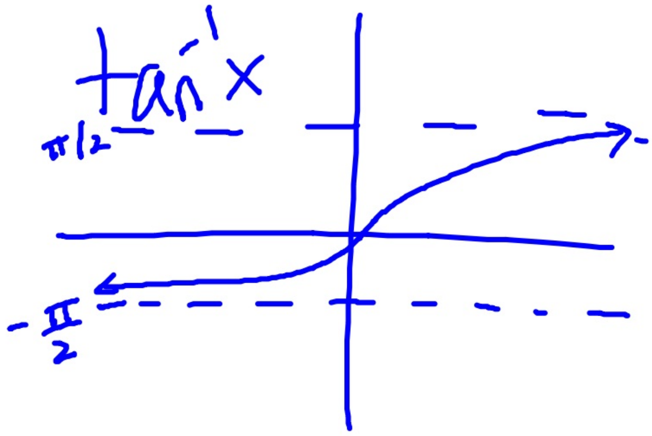
HA: $y=0$

VA: $x=-8$

b) $f(x)=\dfrac{x^2-4}{x-5}$

HA : none

VA: $x=5$



c) $f(x)=\dfrac{5x}{\sqrt{4x^2+1}}$

VA: none

HA : $y=\pm\frac{5}{2}$

$$\lim_{x\to\infty}\frac{5x}{\sqrt{4x^2+1}}=\frac{5}{2}$$

$$\lim_{x\to-\infty}\frac{5x}{\sqrt{4x^2+1}}=-\frac{5}{2}$$

d) $f(x) = \frac{\cos x}{2^x}$

HA: $y = 0$
VA: none

e) $f(x) = \frac{5 \cdot 3^x - 2}{3^x}$

HA: $y = 5$
VA: none

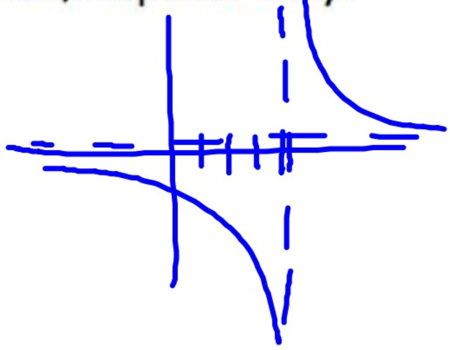
Infinite Limits:

$$\lim_{x \to c^-} f(x) = \infty \qquad \lim_{x \to c^+} f(x) = -\infty$$

ex: Find the limit. If the limit does not exist, explain why.

a) $\lim\limits_{x \to 4} \dfrac{1}{x-4}$ dne

$$\lim_{x \to 4^-} \frac{1}{x-4} \neq \lim_{x \to 4^+} \frac{1}{x-4}$$



b) $\lim_{x \to 4} \frac{1}{(x-4)^2}$

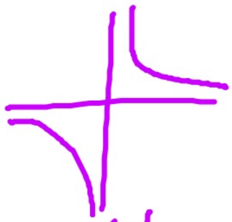
c) $\lim_{x \to 4} \frac{1}{(x-4)^3}$

d) $\lim_{x \to 4} \frac{1}{(x-4)^4}$

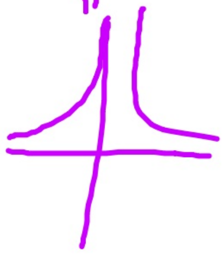
e) $\lim_{x\to 7}\frac{x-9}{x-7}$

dne

$\frac{1}{x}$



$\frac{1}{x^2}$



f) $\lim_{x\to 7}\frac{x-9}{(x-7)^2}$ $= -\infty$

g) $\lim\limits_{x \to 6} \dfrac{x}{x^2 - 36}$

h) $\lim\limits_{x \to 1} \dfrac{x - 1}{x^2 - 7x + 6}$

i) $\lim_{x \to 6} \frac{x-1}{x^2-7x+6}$

j) $\lim_{x \to 2} \frac{x^2+8x+15}{x^2+3x-10}$

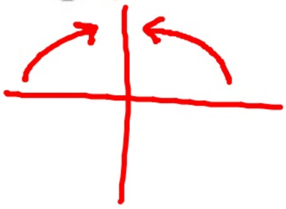
k) $\lim_{x\to 0}\left(x^2 - \frac{1}{x}\right)$ $= \lim_{x\to 0} x^2 - \lim_{x\to 0} \frac{1}{x} =$ dne

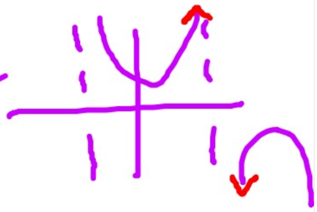
$0 \quad -$ Dne

$= \lim_{x\to 0}\left(\frac{x^3-1}{x}\right) =$ dne

l) $\lim_{x\to\frac{\pi}{2}} -2\sec x$ $\quad -2\lim_{x\to\frac{\pi}{2}} \sec x$ dne

$\pi/2$





$-2\lim_{x\to\frac{\pi}{2}^-} \sec x$

$-\infty$

$-2\lim_{x\to\frac{\pi}{2}^+} \sec x$

$+\infty$

m) $\lim_{x\to -3^-} \frac{x}{\sqrt{x^2-9}}$

In general, if $\lim\limits_{x \to c} f(x) = \frac{n}{0},\ n \neq 0,$ then f(x) must have a

__vertical asymptote__ at x=c.

- If the multiplicity of the factor that produces the vertical asymptote is __odd__, the limit will not exist.

- If the multiplicity of the factor that produces the vertical asymptote is __even__, the limit ~~exists and~~ is either __∞__ or __$-\infty$__.

Limits at Infinity:

$$\lim_{x \to \infty} f(x) \qquad \lim_{x \to -\infty} f(x)$$

end behavior

*The existence or nonexistence of horizontal asymptotes will affect limits at infinity.

**KNOW YOUR LIBRARY OF FUNCTIONS!!!

ex: Find the limit. If the limit does not exist, explain why.

a) $\lim_{x \to \infty} \tan^{-1} x = \frac{\pi}{2}$

b) $\lim_{x\to\infty} \sin x$ dne

c) $\lim_{x\to\infty} \frac{5x^2-4}{x^2+2}$ $= 5$

d) $\lim_{x\to-\infty} \frac{5x^2-4}{x^2+2}$ $= 5$

e) $\lim\limits_{x \to -\infty} \frac{5x-4}{x^2+2}$ = 0

$x \to -\infty$

f) $\lim\limits_{x \to \infty} \frac{5x^2-4}{x+2}$ = ∞

$x \to \infty$

g) $\lim\limits_{x \to \infty} \frac{\sin x}{x}$ = 0

h) $\lim_{x\to\infty} \frac{2^{-x}}{x^2}$

i) $\lim_{x\to\infty} \frac{\sqrt{9x^2+2}}{5x-3}$ $= \frac{3}{5}$

$x \to \infty$

$\frac{5x-1}{2x+7}$

j) $\lim_{x\to-\infty} \frac{\sqrt{9x^2+2}}{5x-3}$ $= -\frac{3}{5}$

$x \to -\infty$

k) $\lim\limits_{x \to \infty} \dfrac{\sqrt{5x^2 - 2x + 3}}{x - 4}$

l) $\lim\limits_{x \to \infty} \dfrac{\sqrt{5x^2 - 2x + 3}}{4 - x}$

Justifying Asymptotes

Horizontal Asymptotes:

If f(x) has a horizontal asymptote at y=c show

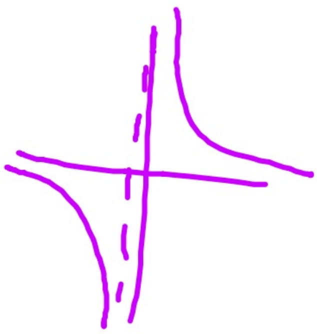
$$\lim_{x \to \infty} f(x) = c$$

or

$$\lim_{x \to -\infty} f(x) = c$$

Vertical Asymptotes:

If f(x) has a vertical asymptote at x=c show



$$\lim_{x \to c^-} f(x) = \infty \text{ or } -\infty$$

or

$$\lim_{x \to c^+} f(x) = \infty \text{ or } -\infty$$

ex: State the horizontal and vertical asymptotes. Then justify your answers using an appropriate limit statement.

a) $f(x) = e^x - 2$

b) $y = \dfrac{\sqrt{6x^2 + 16}}{x - 2}$

ex: If $\lim_{x \to 6^-} f(x) = \infty$, what can be said about the graph of f(x)?