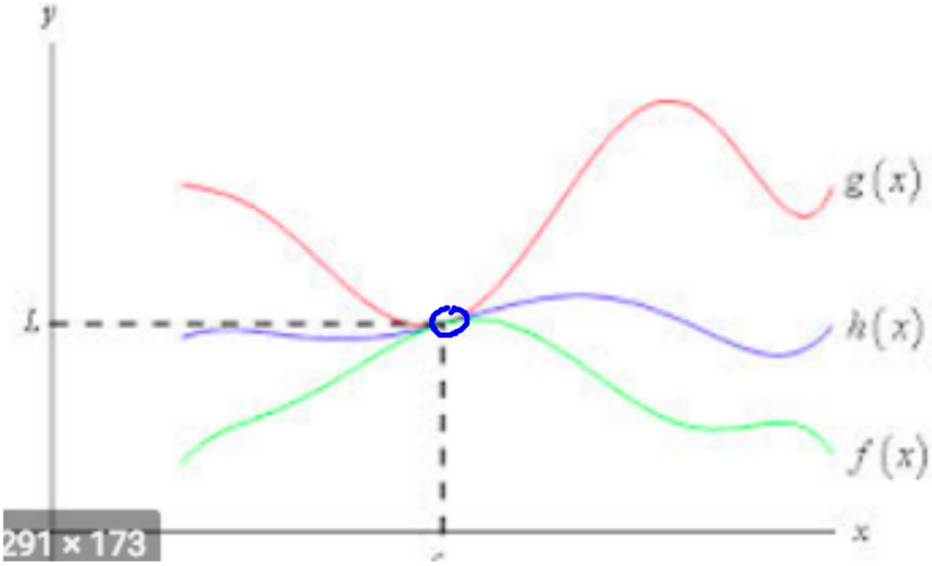
Squeeze Theorem Review



$f(x) \leqslant h(x) \leqslant g(x)$

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1.4 Continuity At A Point & The Intermediate Value Theorem

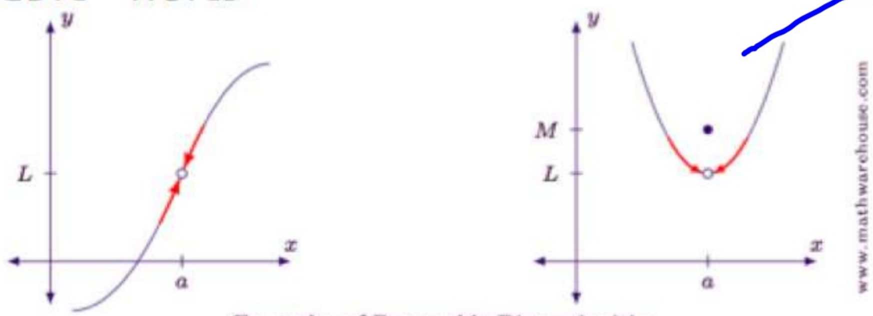
ex: If $f(2)=4$, can you conclude anything about the limit of $f(x)$ as x approaches 2? Explain your reasoning.

ex: If the limit of $f(x)$ as x approaches 2 is 4, can you conclude anything about $f(2)$? Explain your reasoning.

Types of Discontinuities

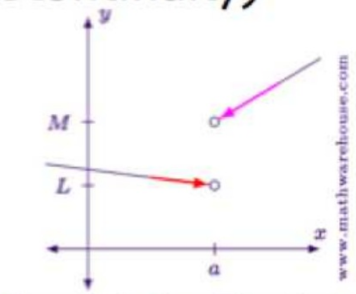
$$\lim_{x\to 2}\frac{x^2-4}{x-2} = \lim_{x\to 2}\frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

- Removable - holes

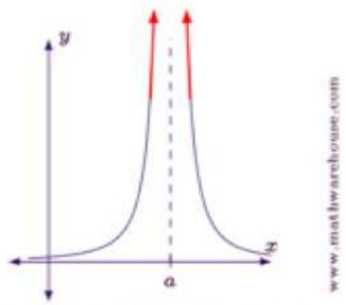


Examples of Removable Discontinuities

- Nonremovable - jumps, vertical asymptotes (a.k.a. infinite discontinuity)



Example of a Jump Discontinuity



Example of an Infinite Discontinuity

ex: At what x-values, if any, is f(x) discontinuous? For each discontinuity state the x-value, the type of discontinuity, and whether the discontinuity is removable or nonremovable.

$$f(x) = \frac{x^2 - 1}{x^2 - 4x + 3}$$

ex: At what x-values, if any, is f(x) discontinuous? For each discontinuity state the x-value, the type of discontinuity, and whether the discontinuity is removable or nonremovable.

$$f(x)=\frac{x^2-1}{x^2-4x+3}=\frac{(x+1)\cancel{(x-1)}}{(x-3)\cancel{(x-1)}}$$

$$f(x)=\frac{x+1}{x-3}$$

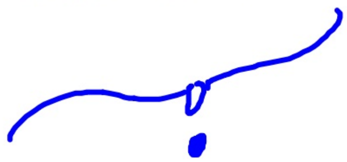
Remov. @ $x=1$ (hole)

nonremov. $x=3$ (VA)

Continuity At A Point, x=c

DEFINITION OF CONTINUITY

Continuity at a Point: A function f is **continuous at c** if the following three conditions are met.



1. $f(c)$ is defined.
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$

Continuity on an Open Interval: A function is **continuous on an open interval (a, b)** if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty, \infty)$ is **everywhere continuous.**

ex: Is f(x) continuous at x=0? Justify your answer.

$$f(x) = \begin{cases} x+1, & x \le 0 \\ x^2+1, & x > 0 \end{cases}$$

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1

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$$

$$1 = 1 = 1$$

ex: Is g(x) continuous at x=3? Justify your answer.

No

$$g(x) = \frac{x^3 - 27}{x - 3} = \frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{(x-3)}}$$

$$\lim_{x \to 3} g(x) \neq g(3)$$

$$\lim_{x \to 3} \underbrace{(x^2+3x+9)}_{27} \neq \underbrace{g(3)}_{\text{undefined}}$$

ex: Find the value of b so that the function f(x) is continuous everywhere. Justify

$$f(x)=\begin{cases} x+3, & x\le 2 \\ bx+7, & x>2 \end{cases}$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$$

$$\lim_{x\to 2^-}(x+3) = \lim_{x\to 2^+}(bx+7) = 5$$

$$5 = 2b+7$$

$$-1 = b$$

ex: Find the value of a so that the function h(x) is continuous everywhere.

Justify

$$h(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & x \neq a \\ 28, & x = a \end{cases}$$

$$\lim_{x \to a} \frac{x^2 - a^2}{x - a} = h(a)$$

$$\lim_{x \to a} (x + a) = 28$$

$$2a = 28 \; ; \; a = 14$$

ex: Find the values of a and b so that the function f(x) is continuous everywhere.

2 connections
2 limits!

$$f(x)=\begin{cases} 2 & x<1 \\ ax^2+bx & 1\le x\le 4 \\ 3 & x>4 \end{cases}$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$$

$$\lim_{x\to 1^-} 2 = \lim_{x\to 1^+}(ax^2+bx)$$

$$2 = a+b$$

ex: Find the values of a and b so that the function f(x) is continuous everywhere.

$$f(x) = \begin{cases} 2 & x < 1 \\ ax^2 + bx & 1 \le x \le 4 \\ 3 & x > 4 \end{cases}$$

2 connections; 2 limit statements

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$-4(2 = a + b)$

$3 = 16a + 4b$

$-8 = -4a - 4b$

$3 = 16a + 4b$

$-5 = 12a$

$a = -\frac{5}{12}$

$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = f(4)$

$16a + 4b = 3$

$2 = a + b$

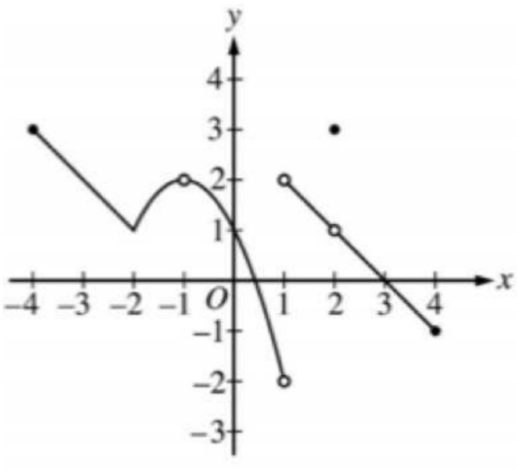
$2 = -\frac{5}{12} + b$

$2 + \frac{5}{12} = b = \frac{29}{12}$

ex: Find the values of a and c so that the function f(x) is continuous everywhere.

$$f(x)=\begin{cases} ax+3 & \text{if } x>5 \\ 8 & \text{if } x=5 \\ x^2+bx+1 & \text{if } x>5 \end{cases}$$

ex:



Graph of f

$\boxed{3}$

The graph of the function f is shown in the figure above. For how many values of x in the open interval $(-4, 4)$ is f discontinuous?

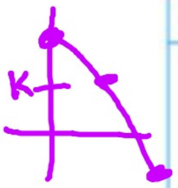
(A) one

(B) two

(C) three

(D) four

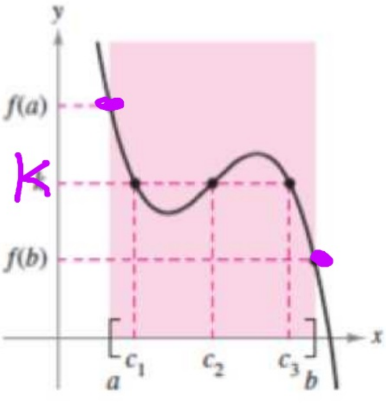
Intermediate Value Theorem (IVT)



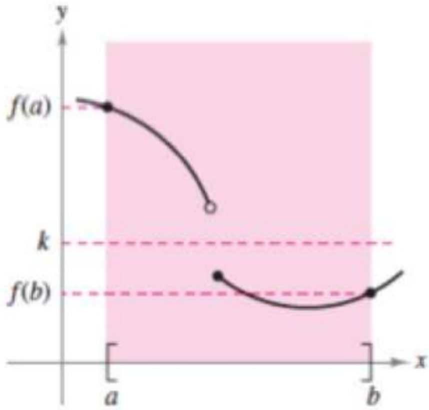
THEOREM 1.13 INTERMEDIATE VALUE THEOREM

If f is continuous on the closed interval $[a, b]$, $f(a) \neq f(b)$, and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a, b]$ such that

$$f(c) = k.$$

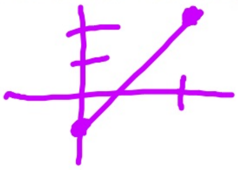


f is continuous on $[a, b]$.
[There exist three c's such that $f(c) = k$.]



f is not continuous on $[a, b]$.
[There are no c's such that $f(c) = k$.]

ex: Use the Intermediate Value Theorem to show a zero exists on f(x) on the given interval.



$$f(x) = x^3 + 2x - 1, \qquad [0,1]$$

$f(0) = -1$
$f(1) = 2$

f(x) is continuous on [0,1] and $f(0) \neq f(1)$ and $f(0) < \cancel{k}\ 0 < f(1)$, by IVT there exists at least one value c such that $f(c) = 0$

ex: Consider the table of values of f(x) given below.

f(x) is continuous on [0,20]

x	0	2	3	10	20
$f(x)$	-2	3	4	20	-10

What is the least amount of time f(x)=15 on [0, 20]?
Justify your answer. 2

1st statement: Since f(x) is cont. on [3,10] and f(3) < $\cancel{x}$ 15 < f(10) by IVT there exists at least one value c such that f(c) = 15 ex:

$h(x) = f(g(x)) - 6$

$h(1) = 3$

$h(3) = -7$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions f and g are continuous for all real numbers. The table above gives values of these functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$. Explain why there must be a value r for $1 < r <$ such that $h(r) = -5$.

$1 < r < 3$

Since h(x) is continuous on [1,3] and h(3) < -5 < h(1) by IVT there exists a value r such that h(r) = -5