73.) $\lim_{x \to 0} \frac{\sin x}{x} \cdot \sin x$

$1 \cdot 0$

0

79) $\lim_{x\to 0} \frac{1-e^{-x}}{e^x-1}$

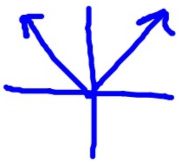
$\lim_{x\to 0} \frac{\left(1-\frac{1}{e^x}\right)e^x}{(e^x-1)e^x} = \lim_{x\to 0} \frac{\cancel{e^x-1}}{e^x\cancel{(e^x-1)}}$

1

1.4 One-Sided Limits

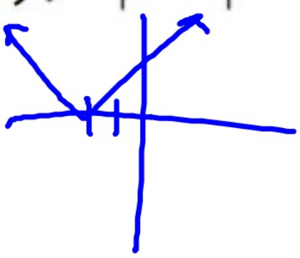
REVIEW: Rewrite each absolute value function as a piecewise function.

a) $y = |x|$



$$y = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

b) $y = |x + 2|$



$$y = \begin{cases} -x-2, & x < -2 \\ x+2, & x \geq -2 \end{cases}$$

c) $y = \frac{|x|}{x}$

$$y = \begin{cases} -1 \ , & x < 0 \\ 1 \ , & x > 0 \end{cases}$$

One-Sided Limits

$$\lim_{x \to c^-} f(x) \qquad\qquad \lim_{x \to c^+} f(x)$$

Left-Sided Limit   Right-Sided Limit

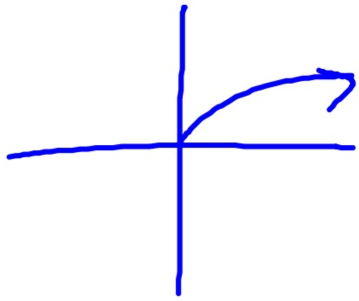
*Use the techniques you learned in 1.2 and 1.3 to find one-sided limits.

ex: Find the limit. If the limit does not exist, explain.

a) $\lim_{x \to 6^-} (x^2 - 21)$ = $36 - 21 = 15$

b) $\lim_{x \to 5^+} \frac{x^2 - 25}{x - 5} = \lim_{x \to 5^+} \frac{\cancel{(x-5)}(x+5)}{\cancel{x-5}} = 10$

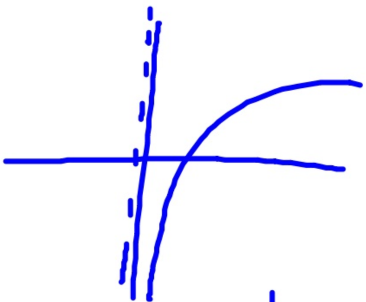
c) $\lim_{x\to 0^-}\sqrt{x}$ dne



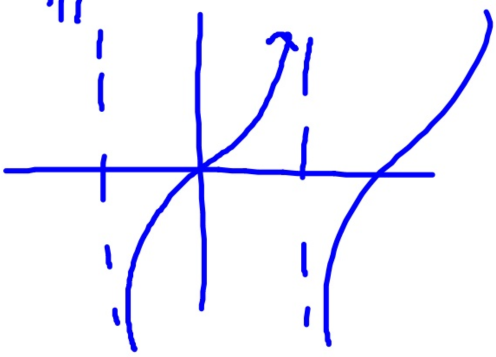
0^- is not in the domain

d) $\lim_{x\to 0^+}\ln x$ $-\infty$ or dne

$x\to 0^+$



e) $\lim_{x\to \frac{\pi}{2}^-}\tan x$ $x\to\frac{\pi}{2}^-$ ∞ or dne



ex: Given f(x) find each limit. If the limit does not exist, explain.

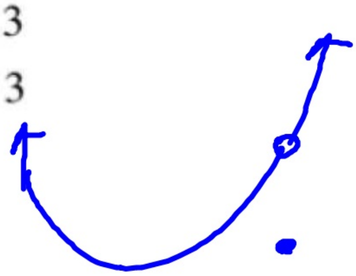
$$f(x)=\begin{cases} x^2+4, & x>5 \\ 2x-3, & x\leq 5 \end{cases}$$

a) $\lim_{x\to 5} f(x)$ dne; $\lim_{x\to 5^-} f(x) \neq \lim_{x\to 5^+} f(x)$

b) $\lim_{x\to 11} f(x) = 125$

ex: Given h(x) find each limit. If the limit does not exist, explain.

$$h(x) = \begin{cases} x^2 - 7, & x \neq 3 \\ 1, & x = 3 \end{cases}$$



a) $\lim\limits_{x \to 3} h(x) = 2$

b) $\lim\limits_{x \to 0} h(x) = -7$

ex: Find the limit. If the limit does not exist, explain.

a) $\lim_{x \to 6} |x-6|$ $= 0$

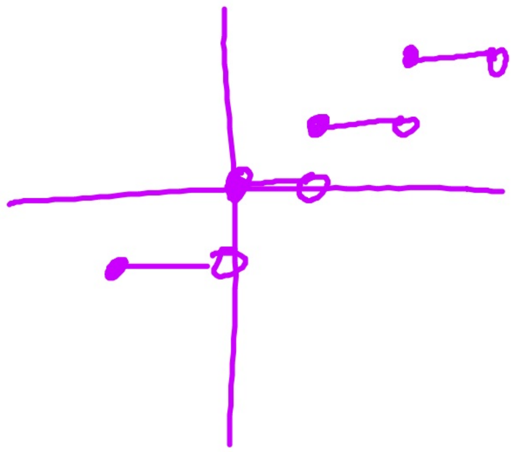
b) $\lim_{x \to 6} \frac{|x-6|}{x-6}$ dne $\lim_{x \to 6^-} \frac{|x-6|}{x-6} \neq \lim_{x \to 6^+} \frac{|x-6|}{x-6}$

c) $\lim_{x \to -5} \frac{|x+5|}{x-3} = 0$

d) $\lim_{x \to 2} [x]$

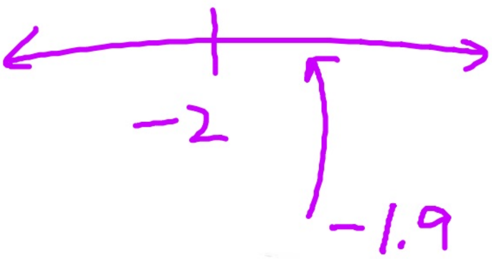
dne

justify...



e) $\lim_{x \to 2.3} [x] = 2$

f) $\lim_{x \to -2^+} [x] = -2$



1976 AB7/BC6

For a differentiable function f, let f^* be the function defined by

$$f^*(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{h}.$$

(a) Determine $f^*(x)$ for $f(x) = x^2 + x$

(b) Determine $f^*(x)$ for $f(x) = \cos x$