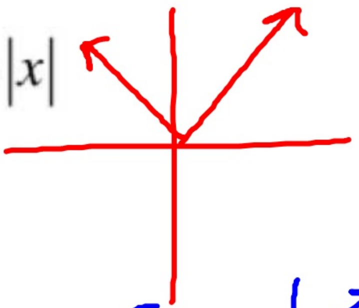
1.4 One-Sided Limits

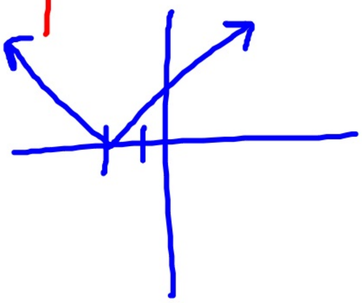
REVIEW: Rewrite each absolute value function as a piecewise function.

a) $y = |x|$



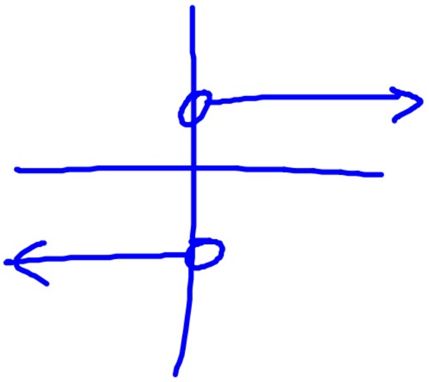
$$y = \begin{cases} x, & x > 0 \\ -x, & x \leq 0 \end{cases}$$

b) $y = |x+2|$



$$y = \begin{cases} -x-2, & x \leq -2 \\ x+2, & x > -2 \end{cases}$$

c) $y = \frac{|x|}{x}$



$$y = \begin{cases} -1, & x < 0 \\ 1, & x > 0 \end{cases}$$

One-Sided Limits

$$\lim_{x \to c^-} f(x) \qquad\qquad \lim_{x \to c^+} f(x)$$

Left-Sided Limit | Right-Sided Limit

$x \to c^-$ | $x \to c^+$

*Use the techniques you learned in 1.2 and 1.3 to find one-sided limits.

ex: Find the limit. If the limit does not exist, explain.

a) $\lim_{x\to 6^-}(x^2-21)$ $= 6^2-21$

$= 36-21$

$= 15$

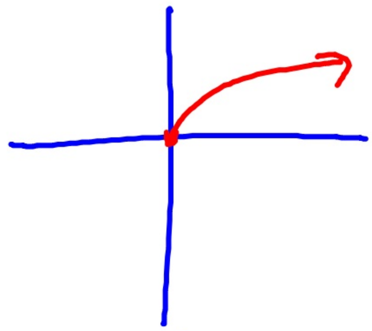
b) $\lim_{x\to 5^+}\frac{x^2-25}{x-5}$ $= \lim_{x\to 5^+}\frac{(x+5)\cancel{(x-5)}}{\cancel{x-5}}$

$= \lim_{x\to 5^+}(x+5)$

$= 10$

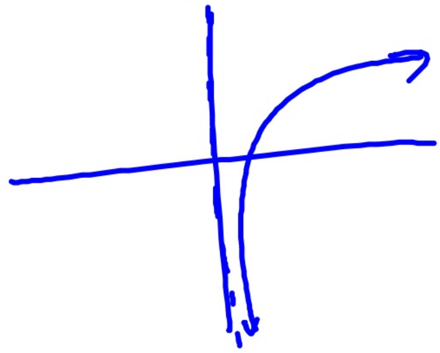
c) $\lim_{x \to 0^-} \sqrt{x}$ dne

$x \to 0^-$



d) $\lim_{x \to 0^+} \ln x$ $= -\infty$ or DNE

$x \to 0^+$



e) $\lim_{x \to \frac{\pi}{2}^-} \tan x$

ex: Given f(x) find each limit. If the limit does not exist, explain.

$$f(x)=\begin{cases} x^2+4, & x>5 \\ 2x-3, & x\le 5 \end{cases}$$

a) $\lim_{x\to 5} f(x)$ dne

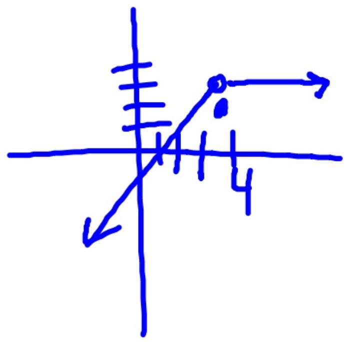
$\lim_{x\to 5^-} f(x) \qquad \lim_{x\to 5^+} f(x)$

$7 \neq 29$

b) $\lim_{x\to 11} f(x)$

ex: Given g(x) find each limit. If the limit does not exist, explain.

$$g(x)=\begin{cases}3, & x>4\\ 2, & x=4\\ x-1, & x<4\end{cases}$$



$$\lim_{x\to 4} g(x) = 3$$

ex: Given h(x) find each limit. If the limit does not exist, explain.

$$h(x) = \begin{cases} x^2 - 7, & x \neq 3 \\ 1, & x = 3 \end{cases}$$

$\cancel{h(3) = 2}$

a) $\lim_{x \to 3} h(x) = 2$

b) $\lim_{x \to 0} h(x)$

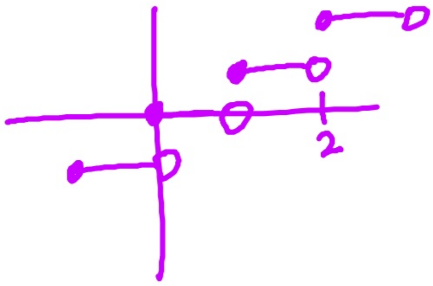
ex: Find the limit. If the limit does not exist, explain.

a) $\lim_{x \to 6} |x-6| = 0$

b) $\lim_{x \to 6} \frac{|x-6|}{x-6}$ dne $\lim_{x \to 6^-} \frac{|x-6|}{x-6} \neq \lim_{x \to 6^+} \frac{|x-6|}{x-6}$

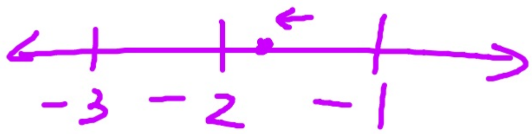
c) $\lim_{x \to -5} \frac{|x+5|}{x-3} = \frac{0}{-8} = 0$

d) $\lim_{x \to 2} [x] = \text{dne}$



e) $\lim_{x \to 2.3} [x] = 2$

f) $\lim_{x \to -2^+} [x] = -2$



g) $\lim_{x \to 3^-} -[5x+6] = -20$

2.9

$-\left[5(2.9)+6\right]$

94) 2x - 4

122) D

79.) $\lim_{x \to 0} \dfrac{1 - \frac{1}{e^x}}{e^x - 1}$

$\lim_{x \to 0} \dfrac{\frac{\cancel{e^x - 1}}{e^x}}{\cancel{e^x - 1}}$

$\lim_{x \to 0} \frac{1}{e^x} = 1$

81.) $\lim_{t \to 0} \frac{\sin(3t)}{2t}$

$\frac{3 \cdot 1}{2} \left[\lim_{t \to 0} \frac{\sin 3t}{3t} \right]$

$\frac{3}{2} \cdot 1$

$\frac{3}{2}$

$\lim_{u \to 0} \frac{\sin u}{u}$

$= 1$

$\sin 2x \neq 2 \sin x$

73.)

$$\lim_{x \to 0} \frac{\sin x \cdot \sin x}{x}$$

$$\lim_{x \to 0} \left(\frac{\sin x}{x}\right) \cdot \sin x$$

$$\lim_{x \to c} f(x)g(x)$$

$$1 \cdot 0$$

$$\lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)$$

$$0$$

63.) $\lim_{h \to 0} \frac{2(x+h)-2x}{h}$

$\lim_{h \to 0} \frac{\cancel{2x}+2\cancel{h}-\cancel{2x}}{\cancel{h}}$

2

61.) $\lim_{x \to 0} \dfrac{\frac{1}{x+3} - \frac{1}{3}}{x}$

$\lim_{x \to 0} \dfrac{3 - (x+3)}{3(x+3)} \cdot \dfrac{1}{x}$

$\lim_{x \to 0} \dfrac{-1}{3(x+3)}$

$-\dfrac{1}{9}$

77.) $$\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\cot x}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\frac{\cos x}{\sin x}}$$

$$\lim_{x \to \frac{\pi}{2}} \cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}}$$

$$1$$

95.) $$\lim_{h \to 0} \frac{\frac{1}{x+h+3} - \frac{1}{x+3}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{x}+\cancel{3} - (\cancel{x}+h+\cancel{3})}{(x+h+3)(x+3)} \cdot \frac{1}{h}$$

$$\lim_{\cancel{h} \to 0} \frac{-1}{(x+h+3)(x+3)}$$

$$\frac{-1}{(x+3)^2}$$

$$65.)\quad \lim_{h\to 0} \frac{\cancel{x^2}+2hx+h^2-\cancel{2x}-2h+\cancel{1}-\cancel{x^2}+\cancel{2x}-\cancel{1}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(2x+\cancel{h}-2)}{\cancel{h}}$$

$$2x-2$$