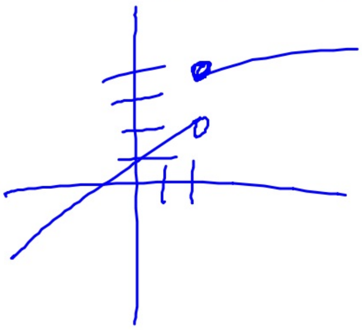
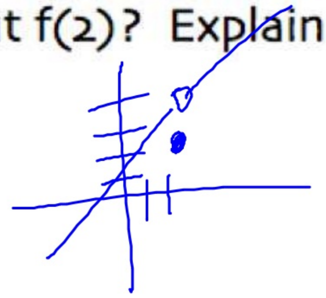
1.4 Continuity At A Point & The Intermediate Value Theorem

ex: If f(2)=4, can you conclude anything about the limit of f(x) as x approaches 2? Explain your reasoning.

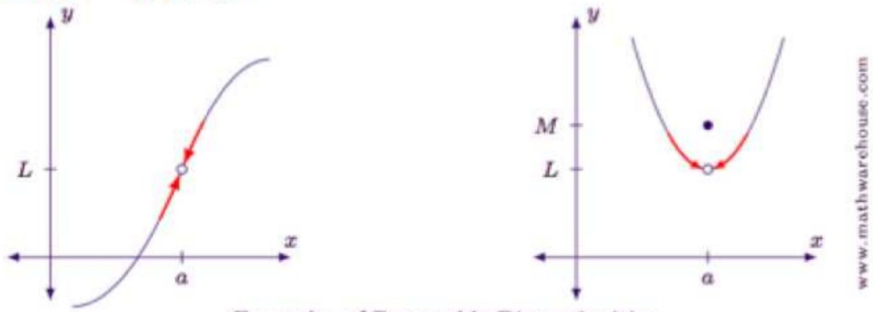


ex: If the limit of f(x) as x approaches 2 is 4, can you conclude anything about f(2)? Explain your reasoning.



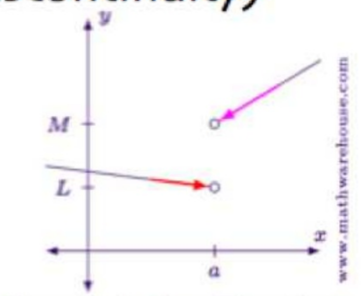
Types of Discontinuities

- Removable - holes

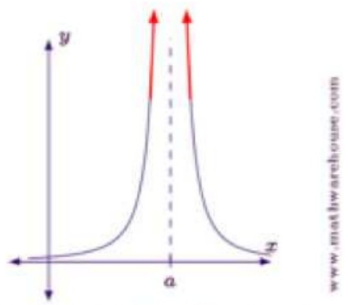


Examples of Removable Discontinuities

- Nonremovable - jumps, vertical asymptotes (a.k.a. infinite discontinuity)



Example of a Jump Discontinuity



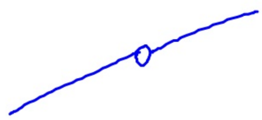
Example of an Infinite Discontinuity

ex: At what x-values, if any, is f(x) discontinuous? For each discontinuity state the x-value, the type of discontinuity, and whether the discontinuity is removable or nonremovable.

$$f(x)=\frac{x^2-1}{x^2-4x+3}=\frac{\cancel{(x-1)}(x+1)}{(x-3)\cancel{(x-1)}}=\frac{x+1}{x-3}$$

Removable discontinuity at x = 1

$\lim_{x\to 1} f(x)$ exists but $\lim_{x\to 1} f(x) \neq f(1)$



Nonremovable discontinuity at x = 3

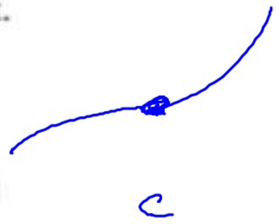
$\lim_{x\to 3} f(x)$ dne

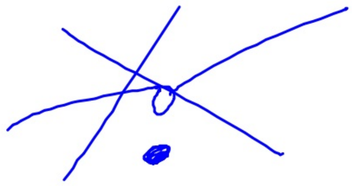
Continuity At A Point, x=c

DEFINITION OF CONTINUITY

Continuity at a Point: A function f is **continuous at** c if the following three conditions are met.

1. $f(c)$ is defined.
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$





Continuity on an Open Interval: A function is **continuous on an open interval** (a, b) if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty, \infty)$ is **everywhere continuous.**

ex: Is f(x) continuous at x=0? Justify your answer.

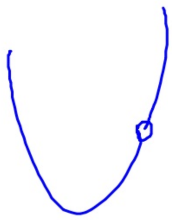
$$f(x) = \begin{cases} x+1, & x \le 0 \\ x^2+1, & x > 0 \end{cases}$$

Yes. $\lim_{x \to 0} f(x) = f(0)$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$$

ex: Is g(x) continuous at x=3? Justify your answer.

$$g(x)=\frac{x^3-27}{x-3}$$



$$g(x)=\frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{x-3}}$$

No.

$\lim\limits_{x\to 3} g(x) \neq g(3)$ or $g(3)$ is undefined ex: Find the value of b so that the function f(x) is continuous everywhere. Justify.

Limits!

$$f(x)=\begin{cases} x+3, & x\le 2 \\ bx+7, & x>2 \end{cases}$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$$

$$\lim_{x\to 2^-}(x+3) = \lim_{x\to 2^+}(bx+7) = 5$$

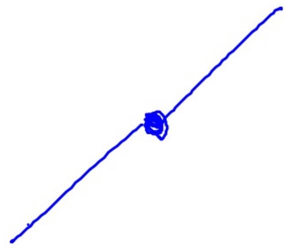
$$\boxed{5 = 2b+7} = 5$$

$$\boxed{b=-1}$$

ex: Find the value of a so that the function h(x) is continuous everywhere.

$$\lim_{x \to a} h(x) = h(a)$$

$$h(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & x \neq a \\ 28, & x = a \end{cases}$$



$$h(x) = \begin{cases} x + a, & x \neq a \\ 28, & x = a \end{cases}$$

$$\lim_{x \to a} (x + a) = 28$$

$$2a = 28$$

$$a = 14$$

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = f(-1)$$

$$\boxed{2 = -a+b} = 2$$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$$

$$\boxed{3a+b = -2} = -2$$

$$\boxed{\begin{aligned} a &= -1 \\ b &= 1 \end{aligned}}$$

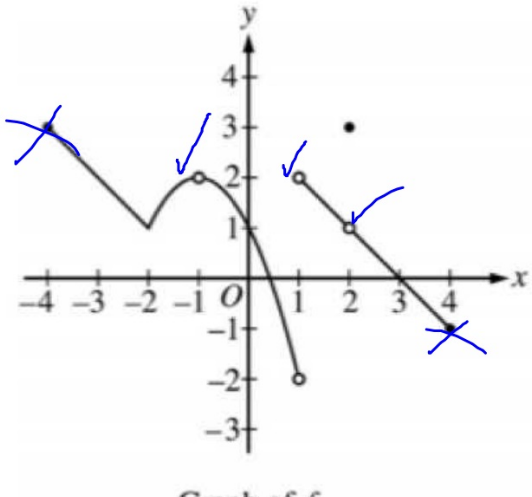
ex: Find the values of a and c so that the function f(x) is continuous everywhere.

$$f(x) = \begin{cases} 2cx^3 - 5ax - 1, & x > -1 \\ 4ax - 1, & x = -1 \\ 5x^2 - 3cx^3 + 4ax - 2, & x < -1 \end{cases}$$

$a = -\frac{8}{27}$

$C = \frac{-4}{3}$

ex:



Graph of f

The graph of the function f is shown in the figure above. For how many values of x in the open interval $(-4, 4)$ is f discontinuous?

(A) one

(B) two

(C) three

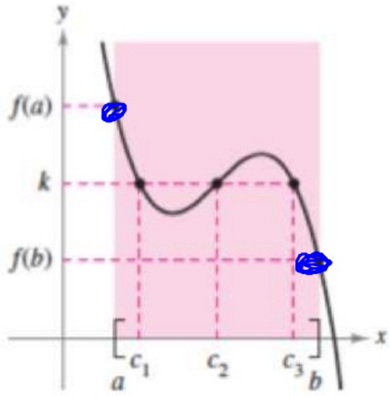
(D) four

Intermediate Value Theorem (IVT)

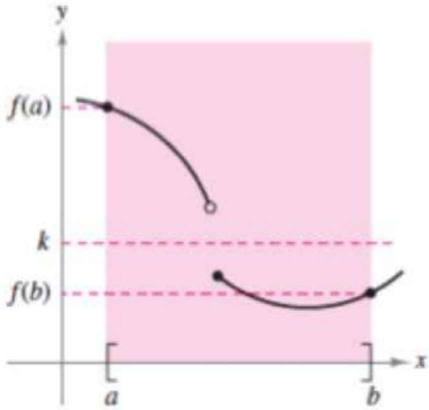
THEOREM 1.13 INTERMEDIATE VALUE THEOREM

If f is continuous on the closed interval $[a, b]$, $f(a) \neq f(b)$, and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a, b]$ such that

$$f(c) = k.$$



f is continuous on $[a, b]$.
[There exist three c's such that $f(c) = k$.]



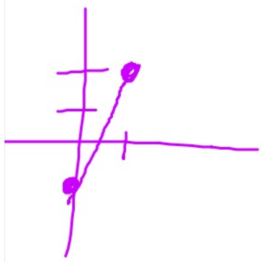
f is not continuous on $[a, b]$.
[There are no c's such that $f(c) = k$.]

ex: Use the Intermediate Value Theorem to show a zero exists on f(x) on the given interval.

$f(0) = -1$

$f(1) = 2$

$$f(x) = x^3 + 2x - 1, \qquad [0,1]$$



$f(x)$ is continuous on $[0,1]$ and $f(0) \neq f(1)$;
since $f(0) < 0 < f(1)$, by IVT there exists
a value c in $[0, 1]$ such that $f(c) = 0$

ex: Consider the table of values of f(x) given below.

f(x)
continuous

x	0	2	3	10	20
$f(x)$	-2	3	4	20	-10

What is the least amount of time f(x)=15 on [0, 20]? Justify your answer.

2 times

ex:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions *f* and *g* are continuous for all real numbers. The table above gives values of these functions and their first derivatives at selected values of *x*. The function *h* is given by $h(x) = f(g(x)) - 6$. Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.