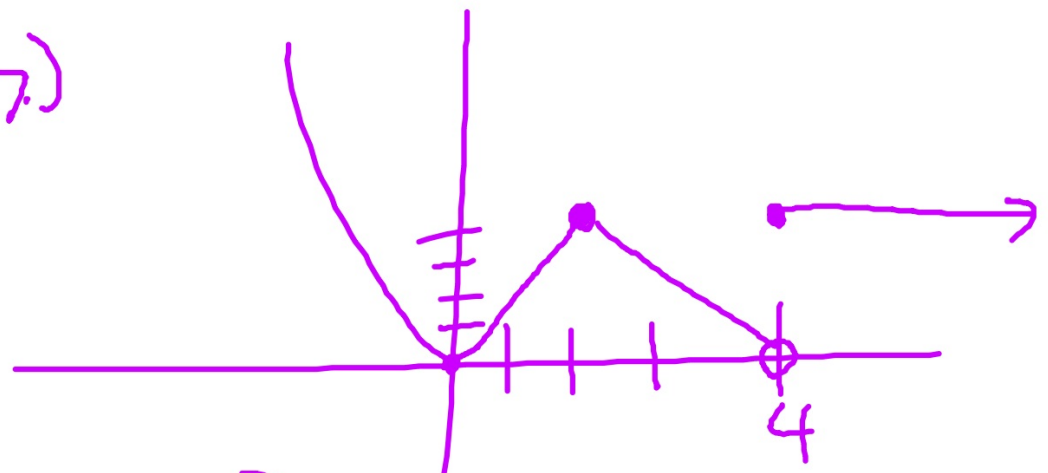


27.)



$\lim_{x \rightarrow c} f(x)$

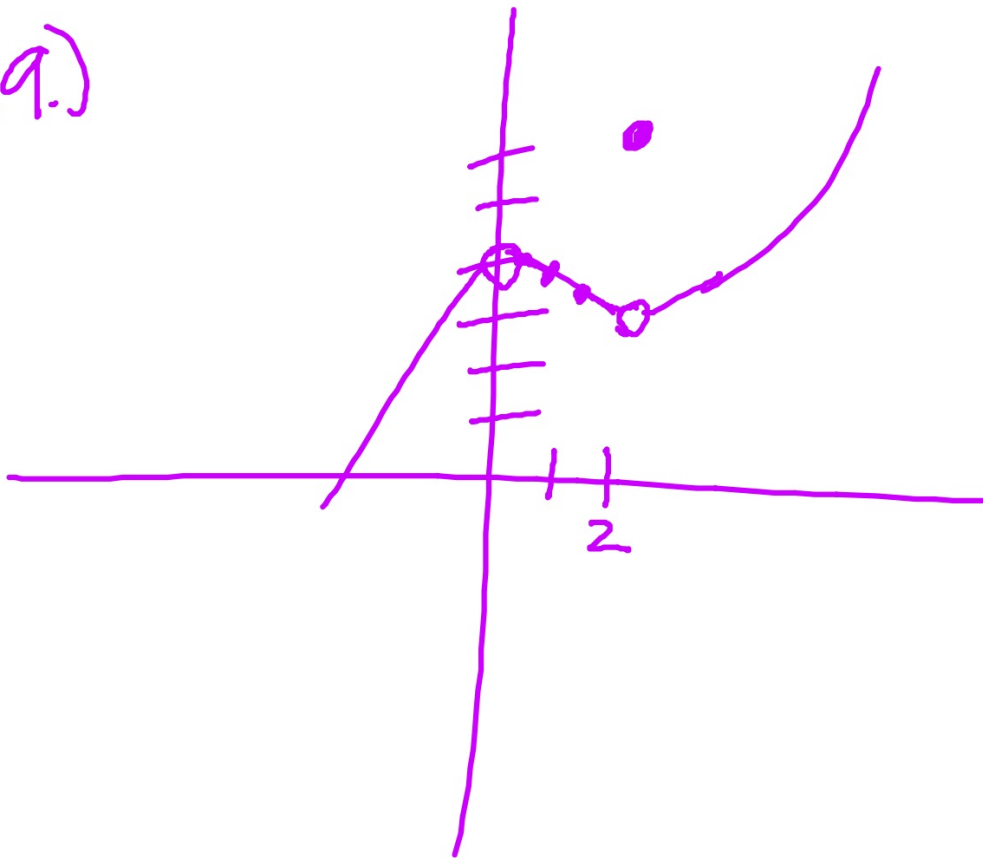
exists except  
where  $c = 4$   
 $\{c | c \neq 4\}$

$$31.) \lim_{x \rightarrow 3} \tan \frac{\pi x}{4} = \tan \frac{3\pi}{4} = -1$$

$$\lim_{x \rightarrow 1} (\ln 3x + e^x)$$

$$\ln 3 + e$$

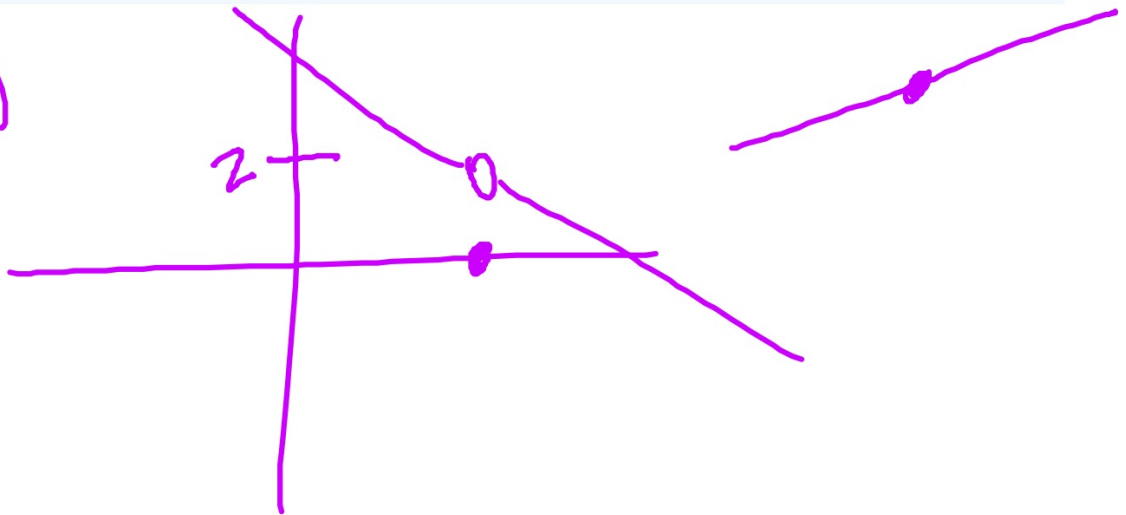
29.)



## 1.3 Evaluating Limits Analytically

- Evaluate a limit using properties of limits.
- Develop and use a strategy for finding limits.
- Evaluate a limit using dividing out and rationalizing techniques.
- Evaluate a limit using the Squeeze Theorem.

19.)



### **THEOREM 1.2 PROPERTIES OF LIMITS**

Let  $b$  and  $c$  be real numbers, let  $n$  be a positive integer, and let  $f$  and  $g$  be functions with the following limits.

$$\lim_{x \rightarrow c} f(x) = L \quad \text{and} \quad \lim_{x \rightarrow c} g(x) = K$$

1. Scalar multiple:  $\lim_{x \rightarrow c} [bf(x)] = bL$
2. Sum or difference:  $\lim_{x \rightarrow c} [f(x) \pm g(x)] = L \pm K$
3. Product:  $\lim_{x \rightarrow c} [f(x)g(x)] = LK$
4. Quotient:  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{L}{K}$ , provided  $K \neq 0$
5. Power:  $\lim_{x \rightarrow c} [f(x)]^n = L^n$

## Strategy for finding limits analytically

1. Direct Substitution
2. Algebraic techniques  
(factoring or rationalizing or simplifying)
3. Special Cases

### THEOREM 1.9 TWO SPECIAL TRIGONOMETRIC LIMITS

$$1. \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad 2. \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

**Ex 1**

$$\lim_{x \rightarrow 2} \frac{\sqrt{x+2}}{x-4}$$

**Ex 2**

$$\lim_{x \rightarrow 7} \sec\left(\frac{\pi x}{6}\right)$$

Ex 3

*Pythagorean Identities*

$$\lim_{x \rightarrow 5\pi/3} \cos x$$

*Other Trig identities*



**Ex 4**

$$\lim_{x \rightarrow 2} \frac{3x^2 - 4x - 4}{2x^2 - 8}$$

**Ex 5**

$$\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 6x + 8}$$

Ex 6

$$\lim_{x \rightarrow \frac{\pi}{3}} \frac{2\cos^2 x + 3\cos x - 2}{2\cos x - 1}$$

Ex 7

$$\lim_{x \rightarrow 0} \frac{\cot x}{\csc x}$$

Ex 8

$$\lim_{x \rightarrow 0} \frac{e^x - e^{2x}}{1 - e^x}$$

Ex 9

$$\lim_{x \rightarrow 16} \frac{\sqrt{x} - 4}{x - 16}$$

Ex 10

$$\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x-3}$$

$$11.) \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{x} - \sqrt{4-x}}$$

Ex 12

$$\lim_{x \rightarrow 4} \frac{\frac{x}{x+1} - \frac{4}{5}}{x-4}$$

Ex 13

$$\lim_{x \rightarrow 0} \frac{3(1 - \cos x)}{x}$$

Ex 14

$$\lim_{t \rightarrow 0} \frac{\sin 3t}{2t}$$

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{11x}$$



Ex 15: Given  $f(x) = 5x - 2$ ,

**find**  $\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$ .

$$16.) f(x) = -x^2 + 4x$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$17.) f(x) = \frac{1}{x}$$

*Analyze a limit with a table*

*Analyze a limit with a graph*

*Analyze a limit analytically  
direct substitution, factoring,  
rationalizing, simplifying,  
special cases*

### Ex. 11

$$\lim_{x \rightarrow c} f(x) = \frac{3}{2}$$

$$\lim_{x \rightarrow c} g(x) = \frac{1}{2}$$

(a)  $\lim_{x \rightarrow c} [4f(x)]$

(b)  $\lim_{x \rightarrow c} [f(x) + g(x)]$

(c)  $\lim_{x \rightarrow c} [f(x)g(x)]$

(d)  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$