1.2 Finding Limits Graphically and Numerically

- What is a limit?

The value that a function approaches as the input, x, approaches some value, y.

- 3 Ways to Find Limits:
    1. Graphically
    2. Numerically
    3. Analytically (Algebraically)

Numerical limit example

$$\lim_{x \to 2} \frac{x-2}{x^2-4}$$

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$						

Graphical limit examples

1) $\lim_{x \to 0} \sec x$

2) $\lim_{x \to 0} \tan x$

Graphical limit examples

3) $\lim_{x\to 0}\frac{|x|}{x}$

4) $\lim_{x\to \pi/2}\tan x$

In general, Limits DO NOT EXIST if:

1. f(x) approaches two different y-values from the left and right.

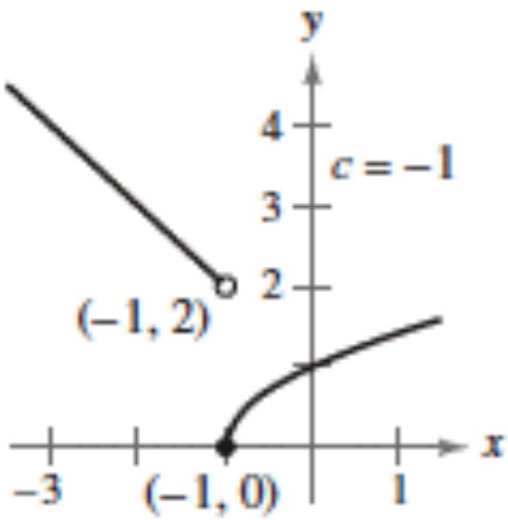
2. f(x) oscillates between two or more y-values.

3. f(x) approaches infinity or negative infinity.*

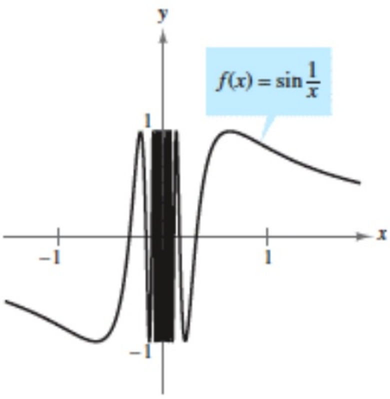
What to write:

If c is finite:	If c is infinite:
$\lim_{x \to c} f(x)$ DNE because	$\lim_{x \to c} f(x)$ DNE because
$\lim_{x \to c^+} f(x) \neq \lim_{x \to c^-} f(x)$	(justify in words)

DNE
Type #1



DNE Type #2



$\lim_{x \to 0} f(x)$ does not exist.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

DNE Type #3

$$\lim_{x \to 5} \frac{2}{x - 5}$$

(a) $f(-2)$

(b) $\lim_{x \to -2} f(x)$

(c) $f(0)$

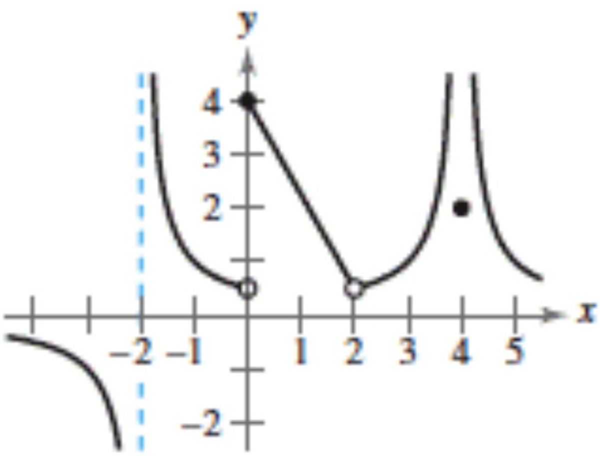
(d) $\lim_{x \to 0} f(x)$

(e) $f(2)$

(f) $\lim_{x \to 2} f(x)$

(g) $f(4)$

(h) $\lim_{x \to 4} f(x)$



Sketching Piecewise functions

Sketch the graph of the following piecewise function.

$$g(x) = \begin{cases} -x^2 + 4 & \text{if } x < 1 \\ 2x - 1 & \text{if } x \geq 1 \end{cases}$$

Sketch the graph of the following piecewise function.

$$h(x)=\begin{cases} x+3 & \text{if } x<-2 \\ x^2 & \text{if } -2\le x<1 \\ -x+2 & \text{if } x\ge 1 \end{cases}$$

Sketch a graph of a function f that satisfies the given values. (There are many correct answers)

$$f(-2) = 0$$

$$f(2) = 0$$

$$\lim_{x \to -2} f(x) = 0$$

$\lim_{x \to 2} f(x)$ does not exist.

. (a) If $f(2) = 4$, can you conclude anything about the limit of $f(x)$ as x approaches 2? Explain your reasoning.

(b) If the limit of $f(x)$ as x approaches 2 is 4, can you conclude anything about $f(2)$? Explain your reasoning.

- Notation

$$\lim_{x\to 5} f(x) = 17 \qquad \lim_{x\to -\infty} e^{x} = 0$$

$$\lim_{x\to 7} = 8$$

1.3: Evaluating limits Analytically (Direct Substitution)

Ex 1

$$\lim_{x \to 2} \frac{\sqrt{x+2}}{x-4}$$

Ex 2

$$\lim_{x \to 7} \sec\left(\frac{\pi x}{6}\right)$$

Ex 3

$$\lim_{x \to 5\pi/3} \cos x$$

$$\lim_{x \to c} f(x) = \frac{3}{2}$$

$$\lim_{x \to c} g(x) = \frac{1}{2}$$

(a) $\lim_{x \to c} [4f(x)]$

(b) $\lim_{x \to c} [f(x) + g(x)]$

(c) $\lim_{x \to c} [f(x)g(x)]$

(d) $\lim_{x \to c} \dfrac{f(x)}{g(x)}$