1.2 Finding Limits Graphically and Numerically

- What is a limit?

The value that a function approaches as the input, x, approaches some value, y.

- 3 Ways to Find Limits:
    1. Graphically
    2. Numerically
    3. Analytically (Algebraically)

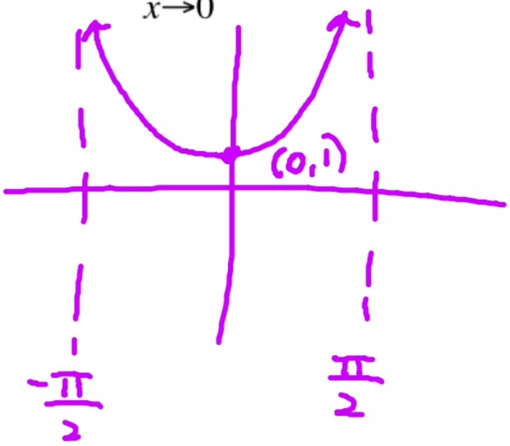
Numerical limit example

$$\lim_{x \to 2} \frac{x-2}{x^2-4} = .25$$

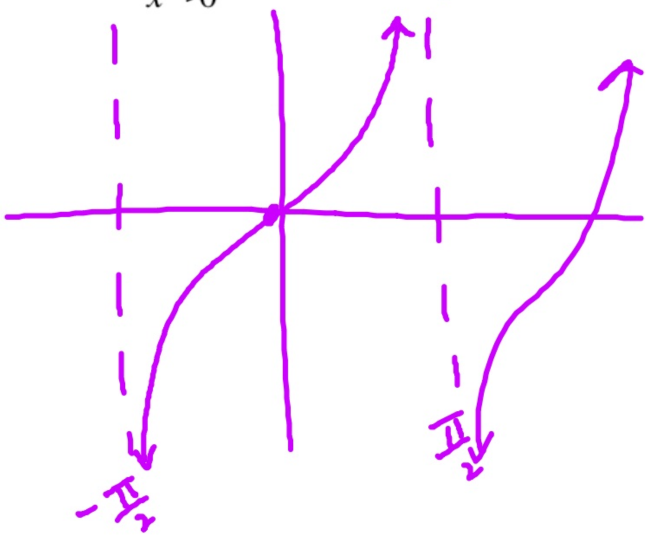
x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	.256	.251	.2501	.24994	.2494	.2439

Graphical limit examples

1) $\lim_{x\to 0} \sec x = 1$

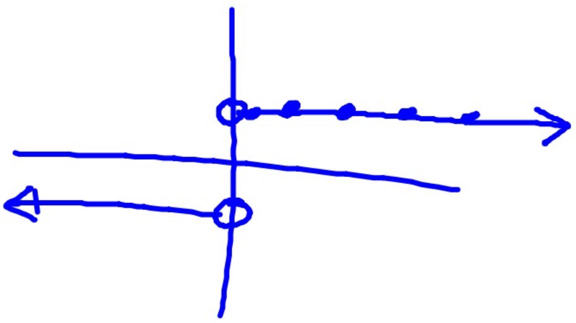


2) $\lim_{x\to 0} \tan x = 0$

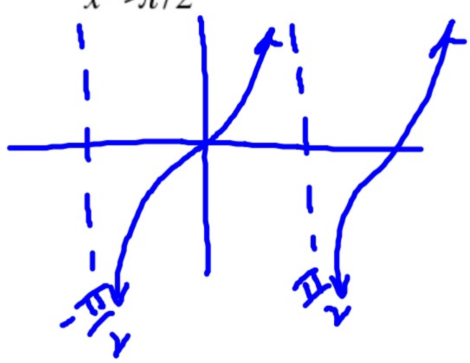


Graphical limit examples

3) $\lim\limits_{x \to 0} \dfrac{|x|}{x}$ dne



4) $\lim\limits_{x \to \pi/2} \tan x$ dne



In general, Limits DO NOT EXIST if:

1. f(x) approaches two different y-values from the left and right.

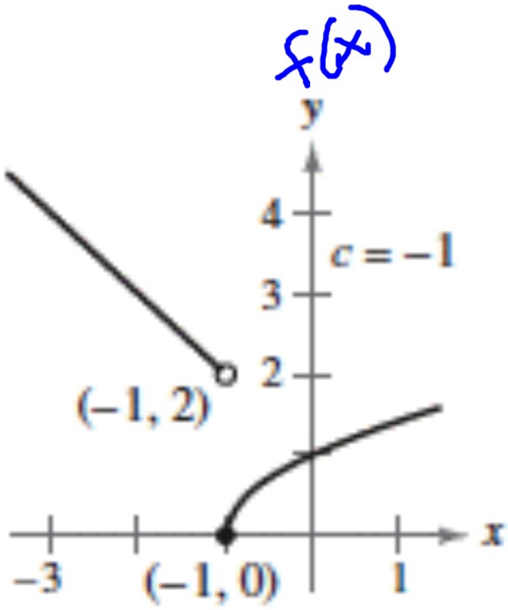
2. f(x) oscillates between two or more y-values.

3. f(x) approaches infinity or negative infinity.*

What to write:

If c is finite:	If c is infinite:
$\lim_{x \to c} f(x)$ DNE because	$\lim_{x \to c} f(x)$ DNE because
$\lim_{x \to c^+} f(x) \neq \lim_{x \to c^-} f(x)$	(justify in words)

DNE
Type #1



$\lim_{x \to -1} f(x)$ dne

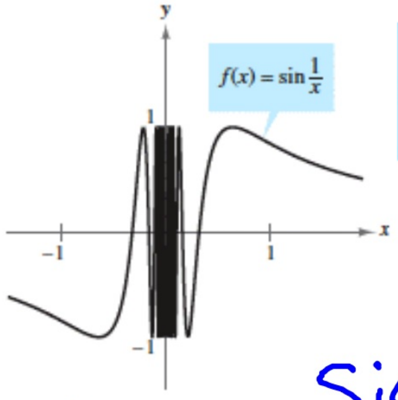
Justify.

$\lim_{x \to -1^-} f(x) \neq \lim_{x \to -1^+} f(x)$

$\underline{2 \neq 0}$

DNE Type #2

$\lim_{x \to 0} f(x)$ dne



$\lim_{x \to 0} f(x)$ does not exist.

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$f(x)$	.544	.506	−.827	.827	−.506	−.544

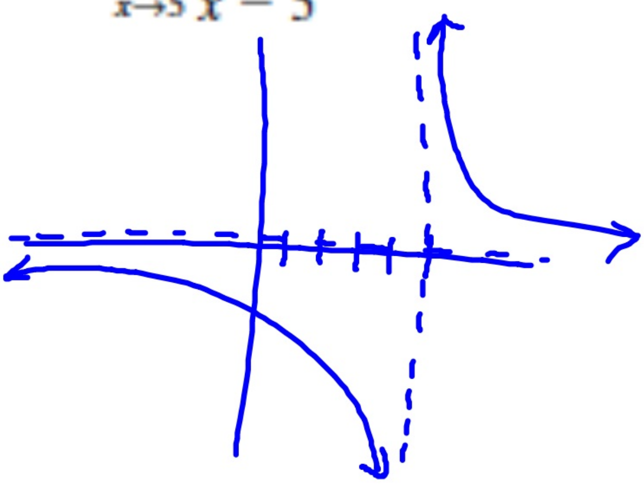
$\sin\left(\frac{1}{x}\right)$

Justify: because $f(x)$ oscillates as $x \to 0$.

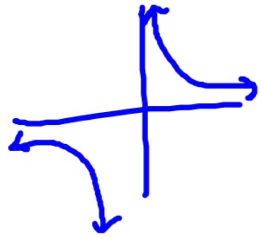
DNE Type #3

$$\lim_{x\to 5} \frac{2}{x-5}$$

dne



$y = \frac{1}{x}$



Justify:

$$\lim_{x\to 5^-} \frac{2}{x-5} \neq \lim_{x\to 5^+} \frac{2}{x-5}$$

$$-\infty \neq \infty$$

(a) $f(-2)$ undefined

(b) $\lim\limits_{x\to -2} f(x)$ dne

(c) $f(0) = 4$

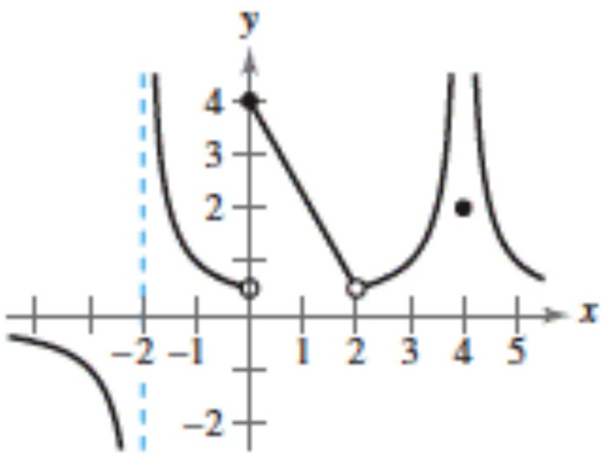
(d) $\lim\limits_{x\to 0} f(x)$ dne

(e) $f(2)$ undefined

(f) $\lim\limits_{x\to 2} f(x) = \frac{1}{2}$

(g) $f(4) = 2$

(h) $\lim\limits_{x\to 4} f(x)$ ∞ or dne

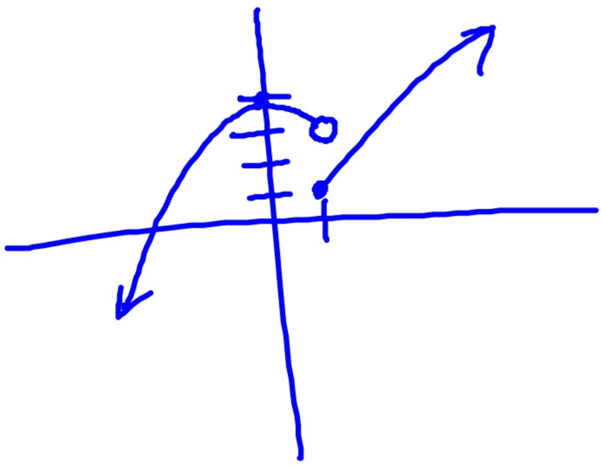


Infinity is a special case of DNE

Sketching Piecewise functions

Sketch the graph of the following piecewise function.

$$g(x)=\begin{cases}-x^2+4 & \text{if } x<1\\ 2x-1 & \text{if } x\geq 1\end{cases}$$



Sketch the graph of the following piecewise function.

$$h(x)=\begin{cases} x+3 & \text{if } x<-2 \\ x^2 & \text{if } -2\le x<1 \\ -x+2 & \text{if } x\ge 1 \end{cases}$$

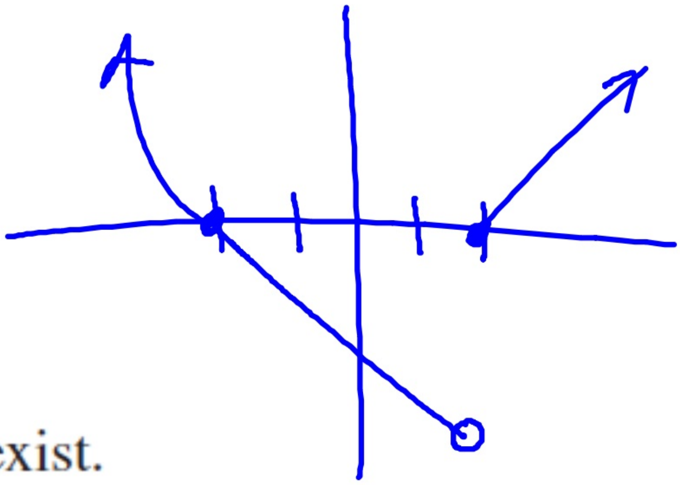
Sketch a graph of a function f that satisfies the given values. (There are many correct answers)

$$f(-2) = 0$$

$$f(2) = 0$$

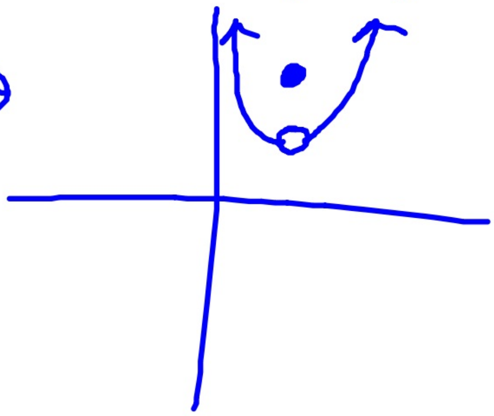
$$\lim_{x \to -2} f(x) = 0$$

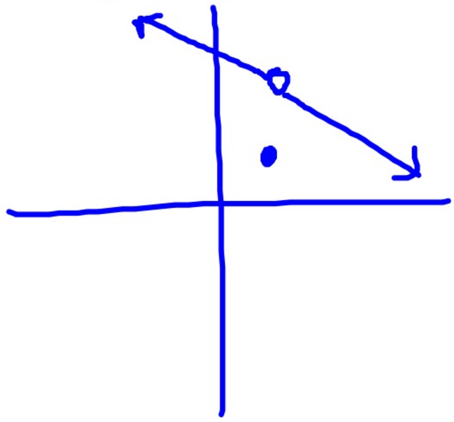
$\lim_{x \to 2} f(x)$ does not exist.



(a) If $f(2) = 4$, can you conclude anything about the limit of $f(x)$ as x approaches 2? Explain your reasoning.

(b) If the limit of $f(x)$ as x approaches 2 is 4, can you conclude anything about $f(2)$? Explain your reasoning.





- Notation

$$\lim_{x\to 5} f(x) = 17 \qquad \lim_{x\to -\infty} e^{x} = 0$$

$$\cancel{\lim_{x\to 7} = 8}$$

1.3: Evaluating limits Analytically (Direct Substitution)

Ex 1

$$\lim_{x\to 2} \frac{\sqrt{x+2}}{x-4} = -1$$

Ex 2

$$\lim_{x\to 7} \sec\left(\frac{\pi x}{6}\right)$$

$$-\frac{2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$$

Ex 3

$$\lim_{x \to 5\pi/3} \cos x = \frac{1}{2}$$

$$\boxed{\begin{aligned}&\lim_{x\to c} f(x) = \tfrac{3}{2}\\ &\lim_{x\to c} g(x) = \tfrac{1}{2}\end{aligned}}$$ given

(a) $\lim_{x\to c} [4f(x)] = 6$

(b) $\lim_{x\to c} [f(x) + g(x)] = 2$

(c) $\lim_{x\to c} [f(x)g(x)] = \frac{3}{4}$

(d) $\lim_{x\to c} \frac{f(x)}{g(x)} = 3$