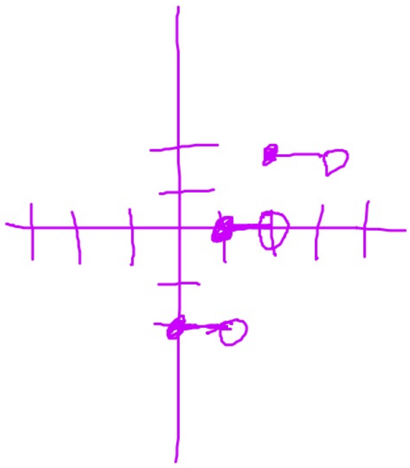
15.) $f(x) = 2[x-1]$

Key point: $(1, 0)$

Length of Bar: $\frac{1}{b} = 1$

Distance: 2
(a)



$\{y \mid y \in 2n, n \in Z\}$

$$h(x) = [x]$$

$$h(-2.4) = -3$$

Symmetry & Even/Odd Functions

ex 1) Evaluate.

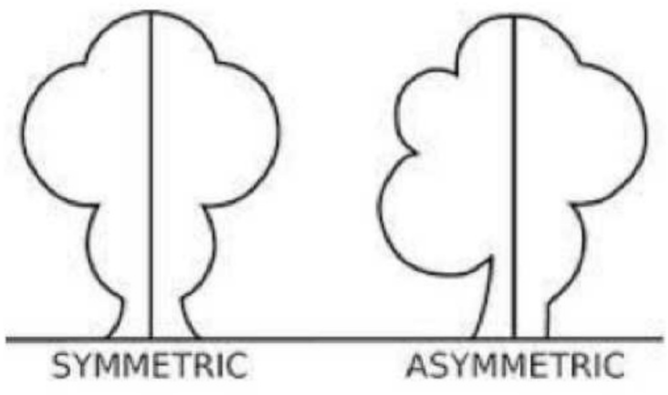
$(-x)^2 = x^2$ $\qquad$ $(-x)^4 = x^4$

$(-x)^3 = -x^3$ $\qquad$ $-3(-x)^3 = 3x^3$

*See printout.

What is Symmetry?

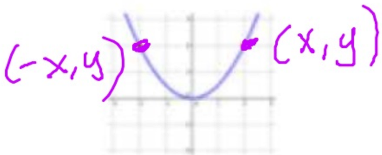
"Symmetry is a recognition of the "matching-ness" of the parts of a shape. In other words, if a graph is reflected across an axis and the graph looks exactly the same as the original, it means that the graph is symmetric with respect to that axis. Graphs can be symmetric to lines and to points.



There are 4 types of symmetry we are concerned with...

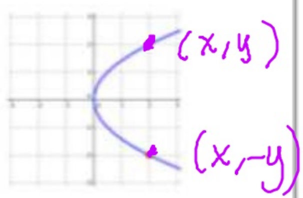
y-axis

A graph has y-axis symmetry if it contains to points (x, y) and (-x, y).



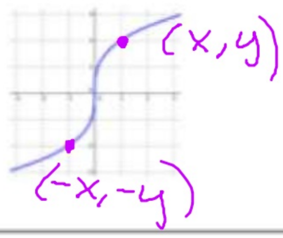
x-axis

A graph has x-axis symmetry if it contains to points (x, y) and (x, -y).



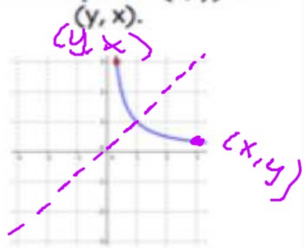
Origin

A graph has origin symmetry if it contains to points (x, y) and (-x, -y).



y=x

A graph has y=x symmetry if it contains to points (x, y) and (y, x).



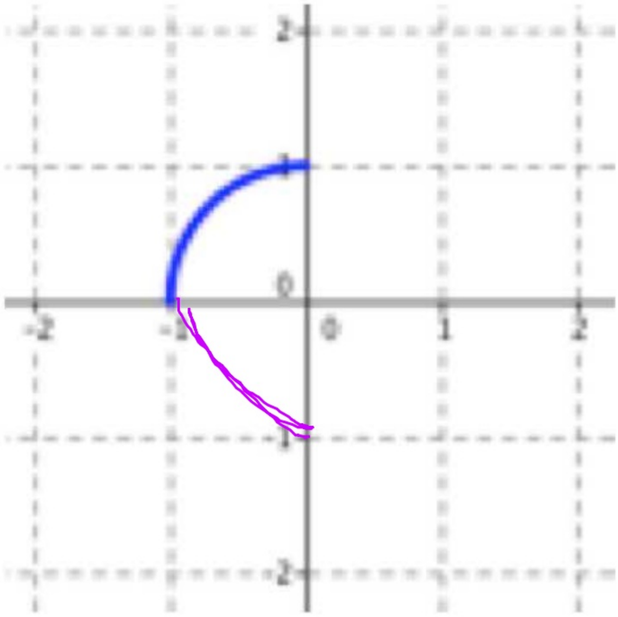
ex 2)

The graph of a curve contains the point (3, -4).

a) If the graph of the curve is symmetric about the y-axis, the graph will also contain the point (-3, -4).	b) If the graph of the curve is symmetric about the x-axis, the graph will also contain the point (3, 4).
c) If the graph of the curve is symmetric about the origin, the graph will also contain the point (-3, 4).	d) If the graph of the curve is symmetric about the line y=x, the graph will also contain the point (-4, 3).

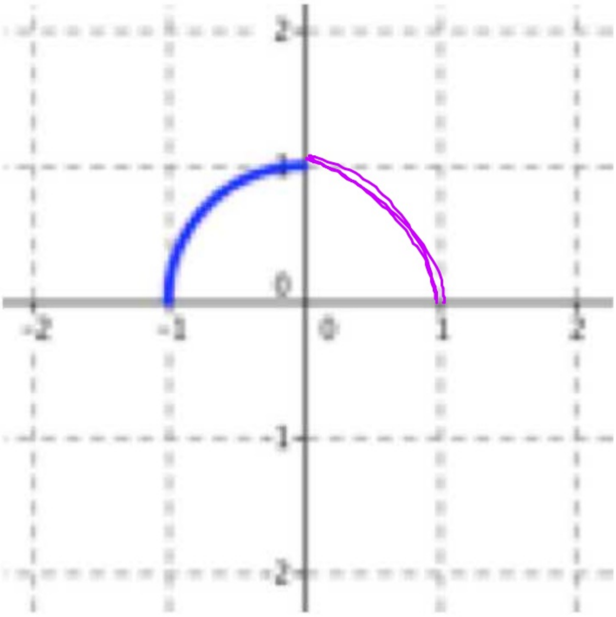
ex 3) The graphs below are portions of complete graphs. Sketch a complete graph for each of the following symmetries: with respect to a) the x-axis, b) the y-axis, c) the origin and d) the line y=x.

a)



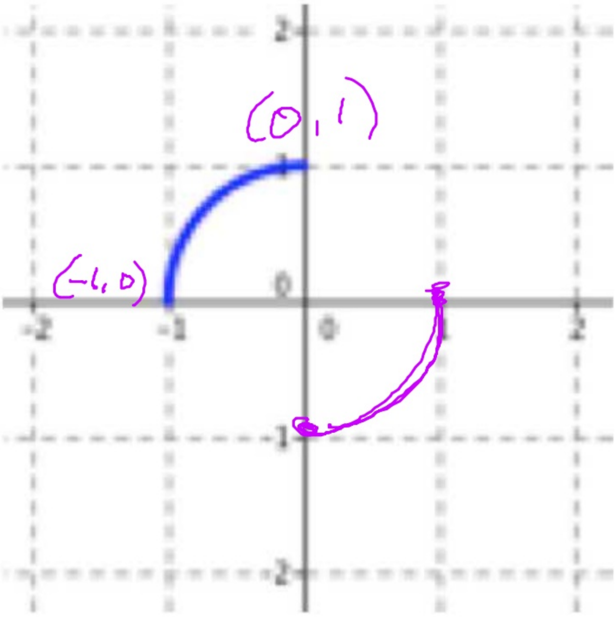
ex 3) The graphs below are portions of complete graphs. Sketch a complete graph for each of the following symmetries: with respect to a) the x-axis, b) the y-axis, c) the origin and d) the line y=x.

b)



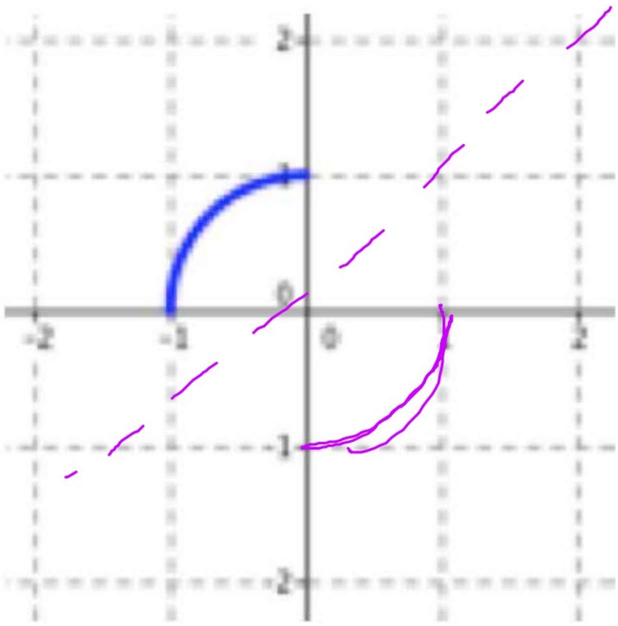
ex 3) The graphs below are portions of complete graphs. Sketch a complete graph for each of the following symmetries: with respect to a) the x-axis, b) the y-axis, c) the origin and d) the line y=x.

c)



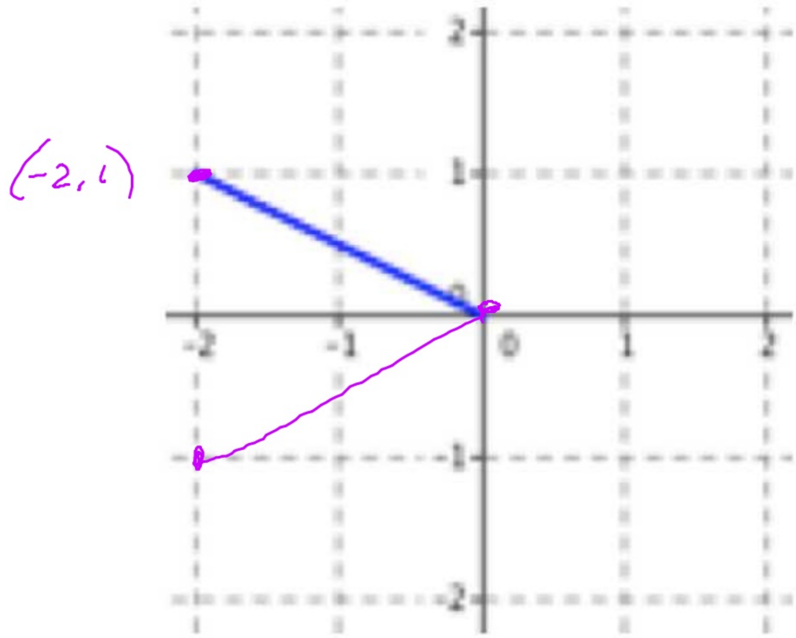
ex 3) The graphs below are portions of complete graphs. Sketch a complete graph for each of the following symmetries: with respect to a) the x-axis, b) the y-axis, c) the origin and d) the line y=x.

d)



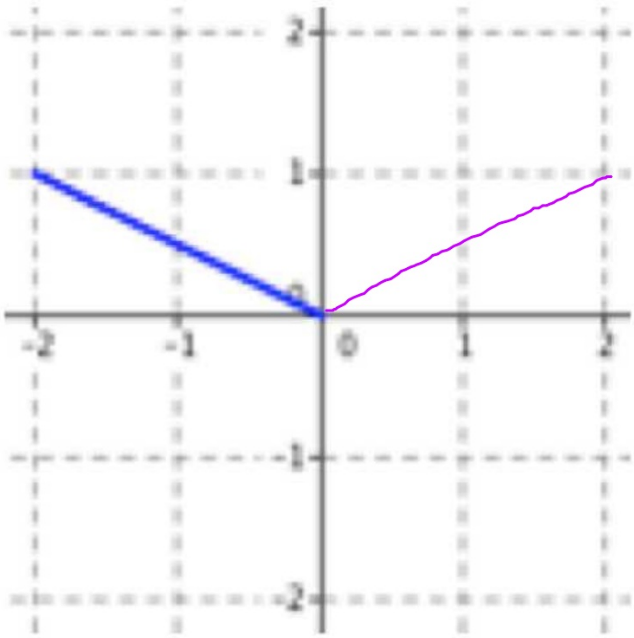
ex 4) The graphs below are portions of complete graphs. Sketch a complete graph for each of the following symmetries: with respect to a) the x-axis, b) the y-axis, c) the origin and d) the line y=x.

a)



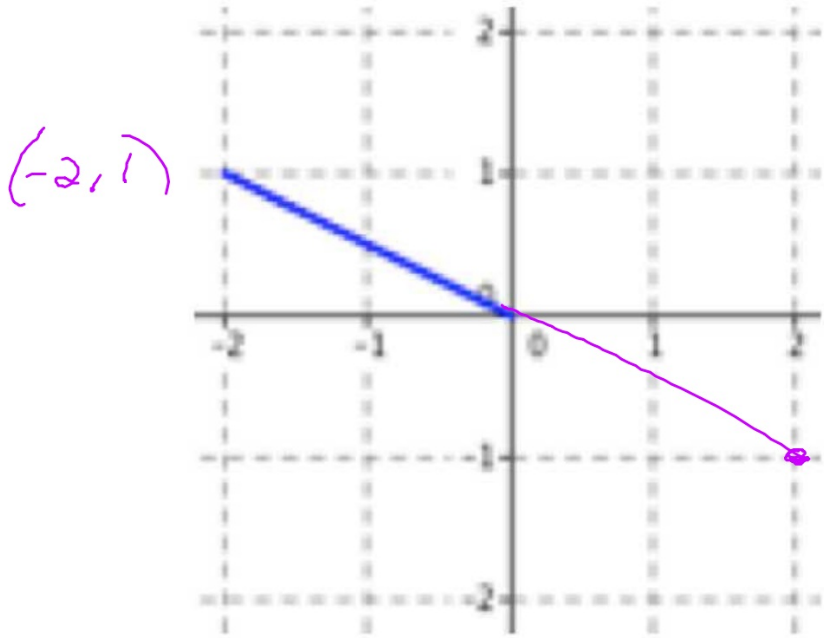
ex 4) The graphs below are portions of complete graphs. Sketch a complete graph for each of the following symmetries: with respect to a) the x-axis, b) the y-axis, c) the origin and d) the line y=x.

b)



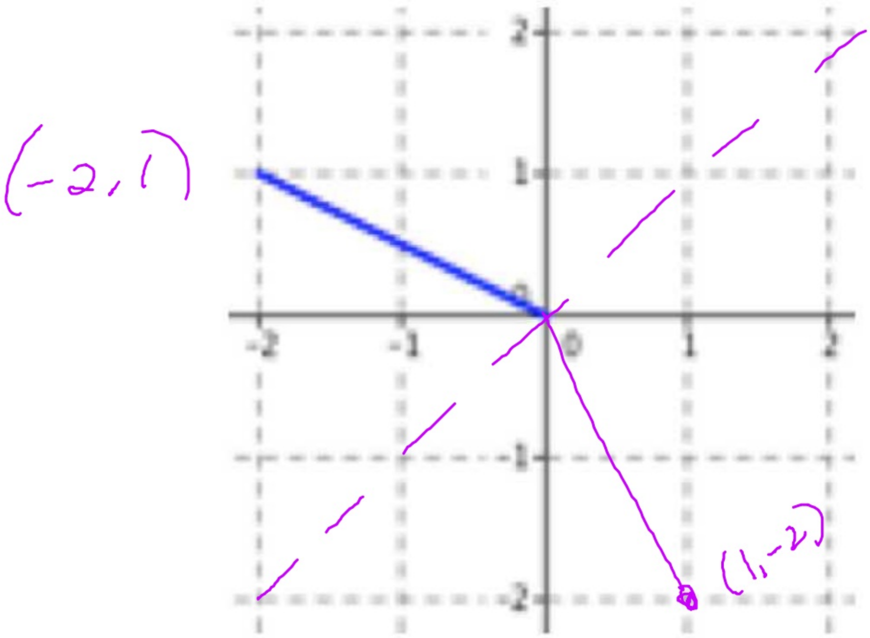
ex 4) The graphs below are portions of complete graphs. Sketch a complete graph for each of the following symmetries: with respect to a) the x-axis, b) the y-axis, c) the origin and d) the line y=x.

c)



ex 4) The graphs below are portions of complete graphs. Sketch a complete graph for each of the following symmetries: with respect to a) the x-axis, b) the y-axis, c) the origin and d) the line y=x.

d)



ex 5) Which library functions have...

a) y-axis symmetry?

ex 5) Which library functions have...

b) x-axis symmetry?

ex 5) Which library functions have...

c) $y=x$ symmetry?

ex 5) Which library functions have...

d) origin symmetry?

Algebraic Tests for Symmetry:

- y-axis: replacing x with –x produces an equivalent equation
- x-axis: replacing y with –y produces an equivalent equation
- origin: replacing x with –x and y with –y produces produces an equivalent equation
- y=x: replacing x with y produces an equivalent equation

ex 6) Determine algebraically if the functions below have y-axis, x-axis, origin or y=x symmetry.

a) $y = 2x^3 - x$

y-axis

No

$y = 2(-x)^3 - (-x)$

$y = -2x^3 + x$

$y = -(2x^3 - x)$

x-axis

No

$-y = 2x^3 - x$

$y = -2x^3 + x$

Origin

Yes!

$-y = 2(-x)^3 - (-x)$

$-y = -2x^3 + x$

$y = 2x^3 - x$

y=x

No

$x = 2y^3 - y$

ex 6) Determine algebraically if the functions below have y-axis, x-axis, origin or y=x symmetry.

b) $y = 4x^4 - x^2 + 5$

x-axis No $-y = 4x^4 - x^2 + 5$ $y = -4x^4 + x^2 - 5$	y-axis Yes $y = 4(-x)^4 - (-x)^2 + 5$ $y = 4x^4 - x^2 + 5$
Origin No $-y = 4(-x)^4 - (-x)^2 + 5$ $y = -4x^4 + x^2 - 5$	$y=x$ No $x = 4y^4 - y^2 + 5$

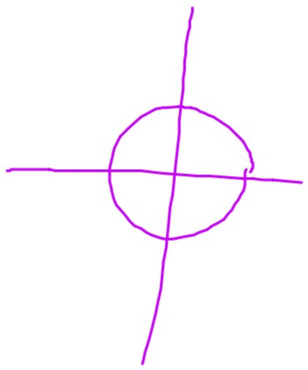
ex 6) Determine algebraically if the functions below have y-axis, x-axis, origin or y=x symmetry.

c) $f(x) = x^3 - 2x^2 + x - 1$

ex 6) Determine algebraically if the functions below have y-axis, x-axis, origin or y=x symmetry.

d) $y = \frac{1}{x}$

x-axis No $-y = \frac{1}{x}$ $y = -\frac{1}{x}$	y-axis No $y = \frac{1}{-x}$
Origin Yes $-y = \frac{1}{-x}$ $y = \frac{1}{x}$	y=x Yes $y \cdot x = \frac{1}{y} \cdot y$ $xy = 1$ $y = \frac{1}{x}$



$$x^2 + y^2 = 1$$

ex 6) Determine algebraically if the functions below have y-axis, x-axis, origin or y=x symmetry.

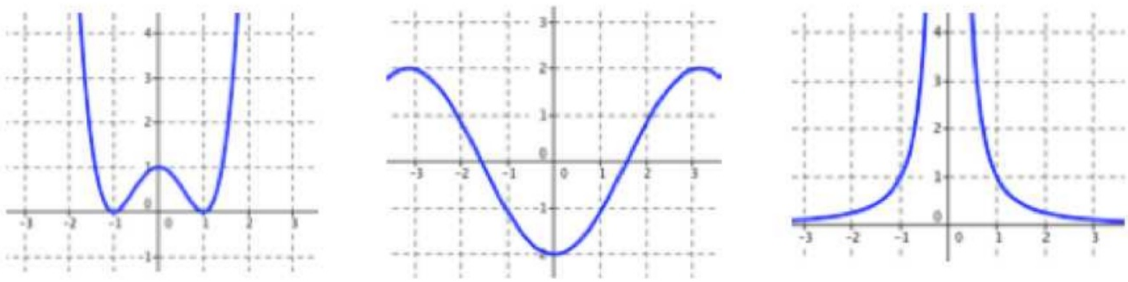
e) $g(x) = \dfrac{5x^3 - x}{x^2 + 4}$

ex 6) Determine algebraically if the functions below have y-axis, x-axis, origin or y=x symmetry.

f) $y = x\sqrt{x^2 - 9}$

x-axis No $-y = x\sqrt{x^2-9}$	y-axis No $y = -x\sqrt{(-x)^2-9}$ $y = -x\sqrt{x^2-9}$
origin Yes $-y = -x\sqrt{(-x)^2-9}$ $y = x\sqrt{x^2-9}$	y=x No $x = y\sqrt{y^2-9}$

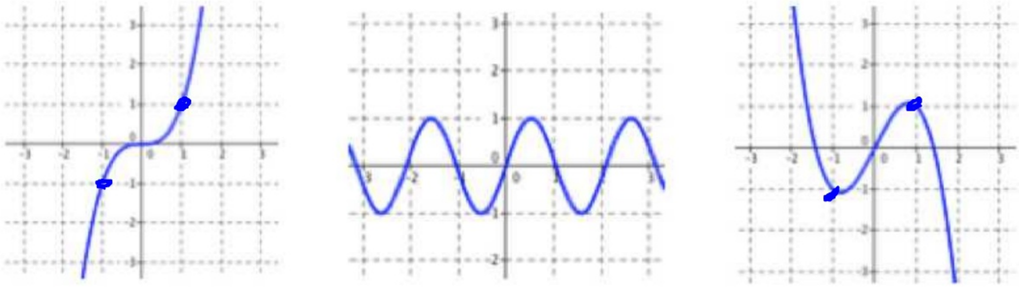
Examples of Even Functions



What do these functions have in common?

end behavior, symmetry with y-axis

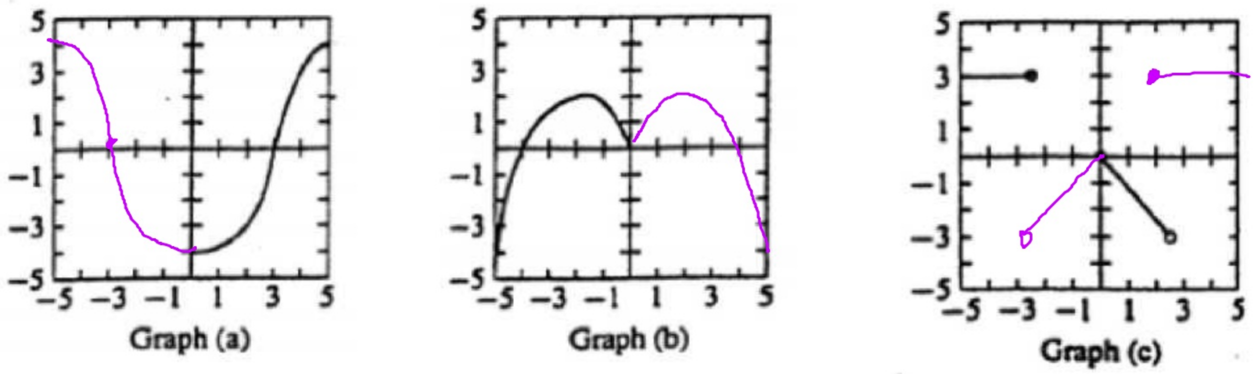
Examples of Odd Functions



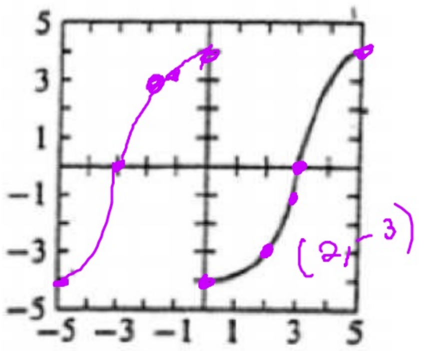
What do these functions have in common?

opposite end behavior / origin symmetry

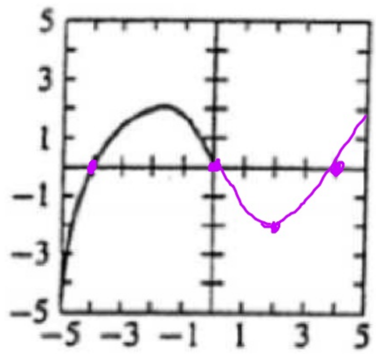
ex 7) Each of the graphs below shows a portion of a graph of an <u>even</u> function over the interval [-5,5]. Complete these graphs.



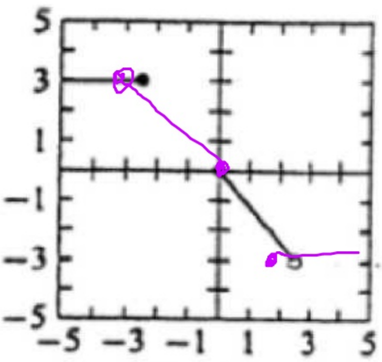
ex 8) Which graph(s) below could be a portion of an <u>odd</u> function over the interval over the interval [-5,5]? Complete these graphs.



Graph (a)



Graph (b)



Graph (c)

ex 9) Table A represents an <u>even</u> function and Table B epresents an <u>odd</u> function. Complete these tables.

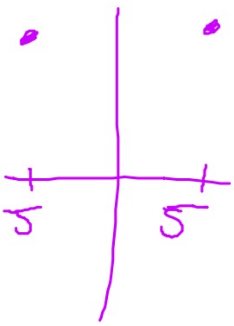
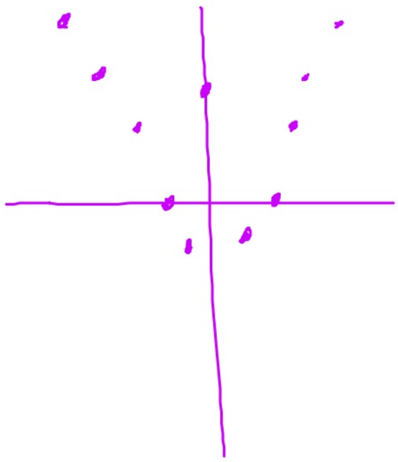
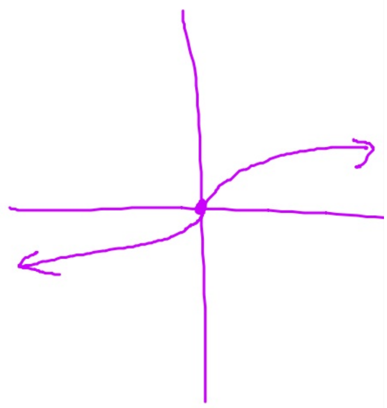


Table A	
-5	8
-4	6
-3	4
-2	0
-1	-1
0	any value
1	−1
2	0
3	4
4	6
5	8



ex 9) Table A represents an <u>even</u> function and Table B epresents an <u>odd</u> function. Complete these tables.

Table B	
-5	-6
-4	3
-3	-2
-2	-5
-1	-3
0	0
1	3
2	5
3	2
4	-3
5	6



ex 10) Which library functions are...

a) even functions?

ex 10) Which library functions are...

b) odd functions?

Algebraic Tests for Even and Odd Functions: $f(x)$

- A function is <u>even</u> if $f(-x) = f(x)$
- A function is <u>odd</u> if $f(-x) = -f(x)$

ex ~~11)~~ Determine algebraically if the given functions are even, odd or neither.

a) $f(x) = 4x^4 - 3x^2 + 2$

$f(-x) = 4(-x)^4 - 3(-x)^2 + 2$

$= 4x^4 - 3x^2 + 2$

even since $f(-x) = f(x)$

ex 11) Determine algebraically if the given functions are even, odd or neither.

b) $f(x) = 3x^2 - 2x + 1$

ex 11) Determine algebraically if the given functions are even, odd or neither.

c) $g(x) = \dfrac{3x}{x^2+7}$

$g(1) = \frac{3}{8}$

$g(-1) = -\frac{3}{8}$

$$g(-x) = \frac{3(-x)}{(-x)^2+7} = \frac{-3x}{x^2+7} = -\left(\frac{3x}{x^2+7}\right)$$

ex 11) Determine algebraically if the given functions are even, odd or neither.

d) $g(x) = \dfrac{4x^3}{x^3 - 2}$

$$g(-x) = \frac{4(-x)^3}{(-x)^3 - 2} = \frac{-4x^3}{(-x)^3 - 2} = \frac{+4x^3}{+(x^3+2)}$$

$-x^3 - 2$

$$= \frac{4x^3}{x^3+2}$$

Neither

ex 11) Determine algebraically if the given functions are even, odd or neither.

even
$h(-x) = h(x)$

e) $h(x) = \dfrac{4x^3}{x^3 - 2x}$

$$h(-x) = \frac{4(-x)^3}{(-x)^3 - 2(-x)} = \frac{-4x^3}{-x^3 + 2x} = \frac{+4x^3}{+(x^3 - 2x)}$$

$$= \frac{4x^3}{x^3 - 2x}$$

ex 11) Determine algebraically if the given functions are even, odd or neither.

f) $G(x) = x\sqrt{x^2 - 1}$

$G(-x) = -x\sqrt{(-x)^2 - 1}$

$G(-x) = -x\sqrt{x^2 - 1}$

Odd since $G(-x) = -G(x)$