4.4 Evaluating Logarithms
4.5 Properties of Logarithms

Midterm Review Chapter 1 due next Tuesday

ex: Evaluate.

a) 2^5 $= 32$

b) $81^{3/4}$ 27

c) $9^{-5/2}$ $\frac{1}{243}$

d) $-16^{5/4}$ -32

ex: Solve.

a) $2^x = 16$ 4

b) $3^x = \frac{1}{3}$ -1

c) $71^x = 1$ 0

d) $25^x = 5$ $\frac{1}{2}$

e) $27^x = 9$ $\frac{2}{3}$

Definition of a Logarithm

Let b and y be positive numbers with $b \neq 1$. The **logarithm of *y* with base *b*** is denoted by $\log_b y$ and is defined as follows:

$$\log_b y = x \quad \text{if and only if} \quad b^x = y$$

The expression $\log_b y$ is read as "log base b of y."

ex: Rewrite in exponential form.

a) $\log_3 9 = 2$

base's exponent

base

$3^2 = 9$

b) $\log_{22} 1 = 0$

$22^0 = 1$

ex: Rewrite in logarithmic form.

a) $3^5 = 243$

$\log_3 243 = 5$

b) $27^{-2/3} = \frac{1}{9}$ $\log_{27} \frac{1}{9} = -\frac{2}{3}$

ex: Evaluate.

a) $\log_4 64 = 3$

$4^x = 64$

b) $\log_3 81 = 4$

$\log_7 1 = 0$

c) $\log_5 25 = 2$

$\log_{11} 1 = 0$

d) $\log_7\left(\frac{1}{7}\right) = -1$

$\log_4 4 = 1$

$\log_7 7 = 1$

e) $\log_{13} 1 = 0$

ex: Evaluate.

f) $\log_{25} 5 = \frac{1}{2}$

g) $\log_5\left(\frac{1}{125}\right) = -3$

h) $\log_{81} 27 = \frac{3}{4}$

i) $\log_2(-4)$

not possible

$2^x = -4$

j) $\log_{25}\left(\frac{1}{5}\right) = -\frac{1}{2}$

k) $\log_{\star}\left(\star^{100}\right) = 100$

$\star > 0, \star \neq 1$

Special Logarithms

SPECIAL LOGARITHMS A **common logarithm** is a logarithm with base 10. It is denoted by $\log_{10}$ or simply by log. A **natural logarithm** is a logarithm with base e. It can be denoted by $\log_e$, but is more often denoted by ln.

Common Logarithm

$$\log_{10} x = \log x$$

Natural Logarithm

$$\log_e x = \ln x$$

Most calculators have keys for evaluating common and natural logarithms.

$e \approx 2.718$

ex: Evaluate.

a) $\log 100 = 2$

b) $\log\left(\frac{1}{10}\right) = -1$

c) $\log .001 = \log \frac{1}{1000} = -3$

ex: Evaluate.

d) $\ln 1 = 0$

$\log_7 1 = 0$

e) $\ln\left(\frac{1}{e}\right) = -1$

$$\boxed{\begin{array}{c} \ln e = 1 \\ \ln 1 = 0 \end{array}}$$

f) $\ln e^2 = 2$

g) $\ln e$

ex: Evaluate on your calculator.

a) $\log 16 = 1.204$

b) $\ln 7 = 1.946$

Logarithms and Exponentials are INVERSES!

$$f(x) = \log_b x \qquad g(x) = b^x$$

$$\boxed{\begin{gathered}\log_4 4^8 = 8 \\ 4^{\log_4 16} = 16\end{gathered}}$$

ex: Evaluate.

a) $(f \circ g)(x) = f(b^x) = \log_b b^x = x$

b) $(g \circ f)(x) = g(\log_b x) = b^{\log_b x} = x$

$$4^{\log_4 16} = 4^2 = 16$$

ex: Evaluate.

a) $7^{\log_7 x}$ $= x$

b) $\log_{62} 62^x$ $= x$

$b^{\log_b x} = x$

c) $\log 10^x$ $= x$

d) $e^{\ln 7}$ $= 7$

$= \log_5 5^{2x} = 2x$

e) $\log_5 25^x$

REVIEW - Exponent Properties

$b^m \cdot b^n =$	b^{m+n}
$\frac{b^m}{b^n} =$	b^{m-n}
$(b^m)^n =$	b^{mn}

Logarithm Properties

Let b, m, and n be positive numbers such that $b \neq 1$.

Product Property	$\log_b mn = \log_b m + \log_b n$
Quotient Property	$\log_b \frac{m}{n} = \log_b m - \log_b n$
Power Property	$\log_b m^n = n \cdot \log_b m$

Logarithm properties are used to EXPAND and CONDENSE logarithmic expressions.

ex: Expand. and simplify

a) $\log_3\left(\frac{abc}{9d}\right) = \log_3 a + \log_3 b + \log_3 c - (\log_3 9 + \log_3 d)$

$= \log_3 a + \log_3 b + \log_3 c - 2 - \log_3 d$

ex: Expand.

b) $\log_5\left(\frac{a^2b^3}{c^4}\right) = \log_5 a^2 + \log_5 b^3 - \log_5 c^4$

$= 2\log_5 a + 3\log_5 b - 4\log_5 c$

c) $\log\left(\frac{100a^2}{b^3c}\right) = \log 100 + \log a^2 - \log b^3 - \log c$

$2 + 2\log a - 3\log b - \log c$

ex: Expand.

$\ln 1 = 0$

d) $\ln\left(\dfrac{1}{ab^2c^3}\right) = \ln 1 - \ln a - \ln b^2 - \ln c^3$

$= -\ln a - 2\ln b - 3\ln c$

*e) $\log_3\left(a+b^2\right)$

can't be expanded (sum!)

$\log_3(ab^2)$

$(x+4)^2 \neq x^2 + 4^2$

ex: Expand.

f) $\log_4\left(\frac{(a+b)}{(a-b^2)}\right) = \log_4(a+b) - \log_4(a-b^2)$

g) $\log_2\left(a^2-b^2\right) = \log_2 (a+b)(a-b)$

$= \log_2(a+b) + \log_2(a-b)$

ex: Expand.

h) $\log_3(a-b)^7 = 7\log_3(a-b)$

i) $\ln\sqrt{\dfrac{y^3+z}{x^3(a+1)^5}} = \frac{1}{2}\ln\left(\dfrac{y^3+z}{x^3(a+1)^5}\right)$

$= \frac{1}{2}\ln(y^3+z) - \frac{3}{2}\ln x - \frac{5}{2}\ln(a+1)$

ex: Condense.

a) $2\log_5 a - 3\log_5 b + 4\log_5(c+d)$

$\log_5 a^2 - \log_5 b^3 + \log_5 (c+d)^4$

$$\log_5\left(\frac{a^2(c+d)^4}{b^3}\right)$$

ex: Condense.

b) $\frac{1}{2}\log x+\frac{3}{2}\log y-10\log z$

$\log x^{1/2}+\log y^{3/2}-\log z^{10}$

$\log\left(\frac{x^{1/2}y^{3/2}}{z^{10}}\right)$ or $\log\left(\frac{\sqrt{xy^3}}{z^{10}}\right)$

ex: Condense.

c) $-3\log x - 4\log y - \frac{2}{3}\log z$

$$\log\left(\frac{1}{x^3 y^4 z^{2/3}}\right)$$