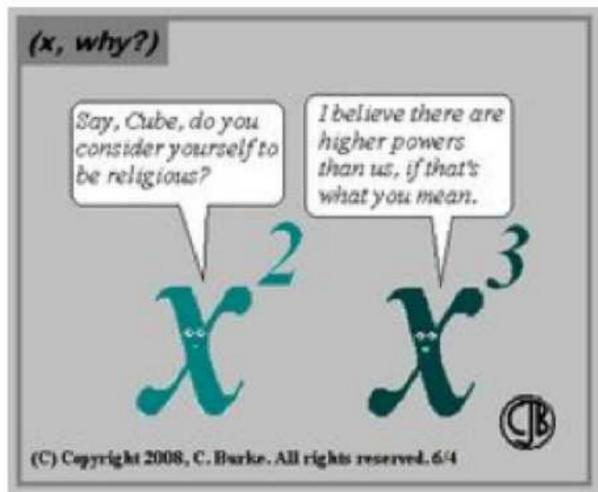


2.1 Properties of Exponents

2.2 Evaluating Polynomial Functions

2.3 Add, Subtract and Multiply Polynomials



REVIEW: Exponent Properties

Property Name	Definition
Product of Powers	$a^m \cdot a^n = a^{m+n}$
Power of a Power	$(a^m)^n = a^{mn}$
Power of a Product	$(ab)^m = a^m b^m$
Negative Exponent	$a^{-m} = \frac{1}{a^m}$
Zero Exponent	$a^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

ex: Simplify the expression.

a) $(x^4)^3 \cdot 2x^4$

$x^{12} \cdot 2x^4$

$2x^{16}$

ex: Simplify the expression.

b) $2x^5 \cdot (5x^4)^{-3}$

$$\frac{2x^5}{(5x^4)^3} = \frac{2(x^5)}{125(x^{12})} = \frac{2}{125x^7}$$

~~XXXXXX~~
~~XXXXXXXXXXXXXX~~

ex: Simplify the expression.

c) $\frac{3x^2y^2}{2x^{-1}(4x^2y)^3}$

ex: Simplify the expression.

d) $25(x-5)^{-2}$

$$\frac{25}{(x-5)^2}$$

$$(x-5)^2 \neq x^2 + 5^2$$

$$\frac{x+4}{10} = \frac{x}{10} + \frac{4}{10}$$

$$\frac{10}{x+4} \neq \frac{10}{x} + \frac{10}{4}$$

Monomial - a number, a variable or a product of numbers and variables

ex.

$$10, -x^4y^3, -4x,$$

Polynomial - an expression involving one or more monomials

$$10, 10+x, x^2+4x-8, 5x^3-11, 3xy^5-x^2$$

Characteristics of Polynomials

1. variables have whole exponents
2. real coefficients
3. no division by variables

ex: Determine whether the expression represents a polynomial.

a) $2x^2 - 4x + \frac{1}{7}$ ✓

b) 0 ✓ $0.2x^2$

c) $\frac{5}{x^2}$ ✗

d) $\frac{x^2}{5} = \frac{1}{5}x^2$ ✓

ex: Determine whether the expression represents a polynomial.

e) \sqrt{x} No

f) $\pi x + i^2 = \pi x - 1$ Yes

g) $3 - ix$ No

h) $\frac{x+1}{x+5} = (x+1)(x+5)^{-1}$ No

ex: Determine whether the expression represents a polynomial.

i) $-3x^3 + \frac{1}{2}x^4$ Yes

$$\sqrt{x} = x^{1/2}$$

j) $3^{1/2}$ Yes

$$3^{1/2} = \sqrt{3}$$

Classifying Polynomials

1. Degree - largest exponent

Degree	Type
0	constant
1	linear
2	quadratic
3	cubic
4	quartic
5	quintic
≥ 6	n^{th} degree

Classifying Polynomials

2. Number of terms

Number of Terms	Type
0	—
1	monomial
2	binomial
3	trinomial
≥ 4	polynomial

ex: Classify the polynomial by the degree and number of terms.

a) $4x - 27x^2 + 3$

quadratic trinomial

b) $\pi x + \pi^2$

linear binomial

ex: Classify the polynomial by the degree and number of terms.

c) $17,000^4$ Constant monomial

$$x^0 = 1$$

d) $(x - 17)^2 = x^2 - 34x + 289$

Quadratic trinomial

ex: Classify the polynomial by the degree and number of terms.

e) $5x^6 + 2x^3 + 4x - 5$

\downarrow
 6^{th} degree polynomial

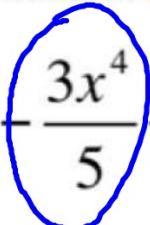
Standard Form of a Polynomial - a polynomial is in standard form when the terms' exponents are in descending order.

ex: Write the polynomial in standard form.

$$1 + 2x - 3x^4$$
$$-3x^4 + 2x + 1$$

Leading Coefficient - the coefficient of the term that defines the degree

ex: Identify the leading coefficient.

$$x - \frac{3x^4}{5} + 10$$


Evaluating Polynomials

There are two ways to evaluate polynomial functions:

1. direct substitution
2. synthetic substitution

Direct Substitution (i.e. "PLUG IN")

ex: Find the indicated polynomial value using direct substitution.

a) $f(x) = x^2 - 5x + 2, \quad f(13) = ?$

$$f(13) = 13^2 - 5(13) + 2$$

$$= 106$$

ex: Find the indicated polynomial value using direct substitution.

b) $g(x) = x^3 + 4x^2 - 1$, $g(6) = ?$

$$g(6) = 6^3 + 4 \cdot 6^2 - 1$$

$$= 216 + 144 - 1$$

$$\boxed{g(6) = 359}$$

Synthetic Substitution - substitution using a chart of coefficients

- *Before using synthetic substitution,
 - the polynomial must be in standard form
 - consider if all terms are present

ex: Find the indicated polynomial value using synthetic substitution.

a) $f(x) = x^2 - 5x + 2, \quad f(13) = ?$ 106

$$\begin{array}{r} 13 \\ | \quad \downarrow \\ 1 & -5 & 2 \\ \hline 1 & 8 & 104 \\ \end{array}$$

A red arrow points from the circled '106' up to the circled '106' in the result column of the synthetic division diagram.

ex: Find the indicated polynomial value using synthetic substitution.

b) $g(x) = x^3 + 4x^2 - 1$, $g(6) = ?$ 359

$$\begin{array}{r} 6 | 1 \ 4 \ 0 \ -1 \\ \downarrow \quad 6 \ 60 \ 360 \\ 1 \ 10 \ 60 \ 359 \end{array}$$

A synthetic division diagram for the polynomial $x^3 + 4x^2 - 1$ at $x = 6$. The divisor is 6. The coefficients 1, 4, 0, -1 are written in blue above the division line. The quotient coefficients 1, 10, 60 are also written in blue below the line. The remainder 359 is circled in red. A red arrow points from the circled 359 up to the red number 359 in the original equation.

Find $f(-3)$ using synthetic substitution.

$$f(x) = -2x^3 + x - 7$$

$$\begin{array}{r} -3 \\ \boxed{-2 \quad 0 \quad 1 \quad -7} \\ \downarrow \qquad \qquad \qquad \qquad \\ \hline -2 \quad 6 \quad -18 \quad 51 \\ \hline \qquad \qquad \qquad \boxed{44} \end{array}$$

$$f(-3) = 44$$

ex: If $f(x) = 3x^2 + bx - 7$ and $f(2) = 15$ find the value of b.

Direct

$$15 = 3(2)^2 + 2b - 7$$

$$15 = 12 + 2b - 7$$

$$15 = 5 + 2b$$

$$5 = b$$

Synthetic

$$\begin{array}{r|rrr} 2 & 3 & b & -7 \\ \downarrow & 6 & & 22 \\ 3 & b+6 & 15 \end{array}$$

$$2(b+6) = 22$$

$$b = 5$$

ex: Consider the four polynomial functions.

$$a(x) = -5$$

$$b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3$$

$$d(x) = 2x - 1$$

Perform the indicated operation.

a) $a(x) + b(x)$

$$-5 + 5x^4 + 2 = 5x^4 - 3$$

$$\begin{aligned} x + x &= 2x \\ x \cdot x &= x^2 \end{aligned}$$

ex: Consider the four polynomial functions.

$$a(x) = -5$$

$$b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3$$

$$d(x) = 2x - 1$$

Perform the indicated operation.

b) $b(x) - c(x) = (5x^4 + 2) - (5x^2 + 4x - 3)$

$$5x^4 + 2 - 5x^2 - 4x + 3$$
$$5x^4 - 5x^2 - 4x + 5$$

ex: Consider the four polynomial functions.

$$a(x) = -5$$

$$b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3$$

$$d(x) = 2x - 1$$

Perform the indicated operation.

$$\begin{aligned} \circ d(x) - 5b(x) &= 2x - 1 - 5(5x^4 + 2) \\ &= 2x - 1 - 25x^4 - 10 \\ &= -25x^4 + 2x - 11 \end{aligned}$$

ex: Consider the four polynomial functions.

$$a(x) = -5$$

$$b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3$$

$$d(x) = 2x - 1$$

Perform the indicated operation.

$$\begin{aligned} \text{d)} \quad a(x)b(x)d(x) &= -5(5x^4 + 2)(2x - 1) \\ &= -5(10x^5 - 5x^4 + 4x - 2) \\ &= -50x^5 + 25x^4 - 20x + 10 \end{aligned}$$

ex: Consider the four polynomial functions.

$$a(x) = -5$$

$$b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3$$

$$d(x) = 2x - 1$$

Perform the indicated operation.

e) $[d(x)]^2$

ex: Consider the four polynomial functions.

$$a(x) = -5$$

$$b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3$$

$$d(x) = 2x - 1$$

Perform the indicated operation.

f) $b(x)c(x)$

$$\begin{array}{r} 5x^2 + 4x - 3 \\ \hline 5x^4 \overline{)25x^6 + 20x^5 - 15x^4} \\ + 2 \overline{)10x^3 + 8x^2 - 6} \\ \hline 25x^6 + 20x^5 - 15x^4 + 10x^3 + 8x^2 - 6 \end{array}$$

ex: Consider the four polynomial functions.

$$a(x) = -5$$

$$b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3$$

$$d(x) = 2x - 1$$

Perform the indicated operation.

g) $c(x)[d(x)]^2$