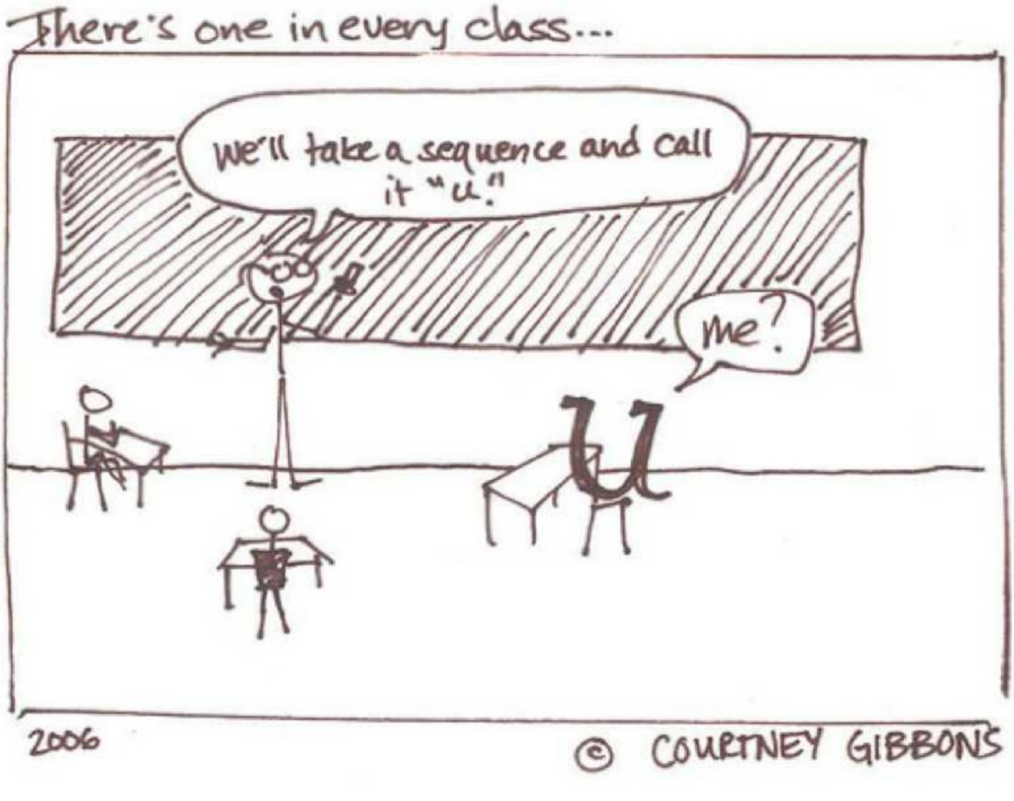
# 7.2 Arithmetic Sequences and Series




*See printout.

## Definition of A Sequence

### Sequences

A **sequence** is a function whose domain is a set of consecutive integers. If a domain is not specified, it is understood that the domain starts with 1. The values in the range are called the **terms** of the sequence.

$$
\begin{array}{lccccccl}
\textbf{Domain:} & 1 & 2 & 3 & 4 & \ldots & n & \text{The relative position of each term} \\
& \downarrow & \downarrow & \downarrow & \downarrow & & \downarrow & \\
\textbf{Range:} & a_1 & a_2 & a_3 & a_4 & \ldots & a_n & \text{Terms of the sequence}
\end{array}
$$

A *finite sequence* has a limited number of terms. An *infinite sequence* continues without stopping.

**Finite sequence:** 2, 4, 6, 8 **Infinite sequence:** 2, 4, 6, 8, . . .

A sequence can be specified by an equation, or *rule*. For example, both sequences above can be described by the rule $a_n = 2n$

## Definition of A Series

**Series**

When the terms of a sequence are added together, the resulting expression is a **series**. A series can be finite or infinite.

**Finite series:** $2 + 4 + 6 + 8$ **Infinite series:** $2 + 4 + 6 + 8 + \cdots$

*In other words, a series is the sum of a sequence.

Notation

$n$ "term #"

$a_n$ "$n^{th}$ term"

Types of Sequences & Series

- Arithmetic
- Geometric
- etc.

Rule

A rule is a formula used to generate the terms of a sequence or series

Rules can be explicit or recursive. For example:

- Explicit: $a_n = 7n - 1$

$a_2 = a_1 - 2$

- Recursive: $a_1 = 3$, $a_n = a_{n-1} - 2$

previous term

ex: Find $a_{\cancel{5}\,4}$.

a) $a_n = 7n - 1$ (explicit)

$a_4 = 7(4) - 1$

$= 27$

b) $a_1 = 3$, $a_n = a_{n-1} - 2$ (recursive)

previous term

$a_2 = a_{2-1} - 2 = 3 - 2 = 1$

$a_3 = a_{3-1} - 2 = 1 - 2 = -1$

$a_4 = a_{4-1} - 2 = -1 - 2 = -3$

# Arithmetic Sequences

In an **arithmetic sequence**, the difference of consecutive terms is constant. This constant difference is called the **common difference** and is denoted by *d*.

ex: Determine if the sequences is arithmetic. If so, identify the common difference.

a) 1, 2, 3, 4, 5 . . . Yes

$d = 1$

b) 1, 1, 2, 3, 5 . . .

no

c) 3, 0, -3, -6, -9 Yes

$d = -3$

$a_n = 1 - 2n$

$a_1 = -1 \qquad (1, -1)$

$a_2 =$

$a_3 =$

$a_4 =$

$a_5 =$

ex: Write the 1st 5 terms of the sequence and sketch the graph.

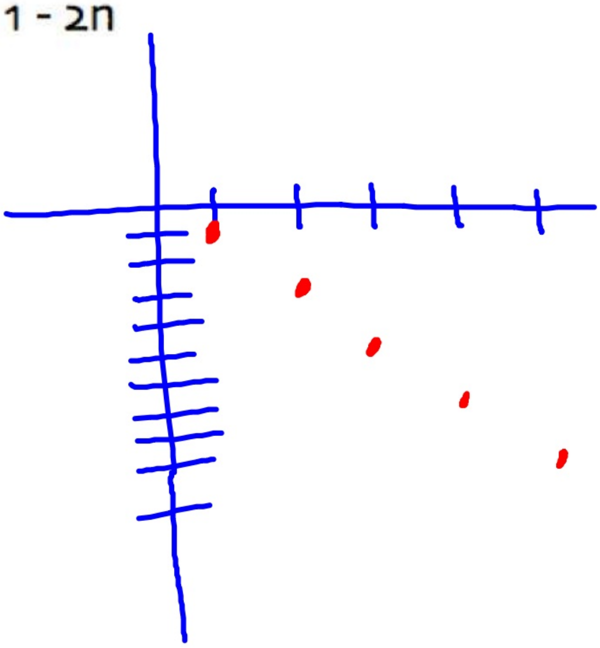
$a_n = 1 - 2n$

$a_1 = -1$

$a_2 = -3$

$\vdots$

$a_5 = -9$



## Writing Explicit Rules for Arithmetic Sequences/Series

*Since arithmetic sequences have a linear pattern, the explicit rule is linear!

Recall Point-Slope: $y - y_1 = m\,(x - x_1)$

Remember:

$n = x$

$a_n = y$

$d = m$

$$\boxed{\text{Explicit Rule: } a_n - a_\# = d\,(n - n_\#)}$$

ex: Write an explicit rule for the arithmetic sequence.

$a_n - a_\# = d(n - n_\#)$

a) 20, 30, 40, 50 . . .

$a_1 = 20$

$d = 10$

$a_n - 20 = 10(n-1)$

$\boxed{a_n = 10n + 10}$

b) 2, -2, -6, -10, -14 . . .

$a_1 = 2$

$d = -4$

$a_n - 2 = -4(n-1)$

$a_n = -4n + 6$

ex: Write an excplit rule for the arithmetic sequence.

c) $a_6 = 7$, $d = 9$

$$a_n - 7 = 9(n-6)$$

$$a_n = 9n - 47$$

ex: Write an excplit rule for the arithmetic sequence.

d) $a_{10} = -5$, $a_{20} = 75$

$(10, -5)$

$(20, 75)$

$m = \frac{75+5}{20-10}$

$d = 8$

$a_n - (-5) = 8(n - 10)$

$a_n = 8n - 85$

Writing Recursive Rules for Arithmetic Sequences/Series

*Recursive rules give the beginning term or terms of a sequence and an equation that shows how $a_n$ is related to one or more previous terms.

ex: Write a recursive rule for the arithmetic sequence.

a) 20, 30, 40, 50 . . .

$$\boxed{\begin{array}{l} a_1 = 20 \\ a_n = a_{n-1} + 10 \end{array}}$$

previous term

ex: Write a recursive rule for the arithmetic sequence.

b) 2, -2, -6, -10, -14 . . .

$$\boxed{\begin{array}{l} a_1 = 2 \\ a_n = a_{n-1} - 4 \end{array}}$$

The Sum of a FINITE Arithmetic Sequences

**The Sum of a Finite Arithmetic Series**

The sum of the first $n$ terms of an arithmetic series is:

$$S_n = n\left(\frac{a_1 + a_n}{2}\right) = \frac{n}{2}(a_1 + a_n)$$

$S_n$ sum of the $1^{st}$ n terms

n number of terms in the sum

$a_1$ $1^{st}$ term in the sequence

$a_n$ last term in the sequnce

*Infinite arithmetic sequences have "no sum." In other words the sum of an infinite arithmetic sequence is infinity or negative infinity and the sum diverges.

*Gauss*

$1+2+3+\ldots\ldots+98+99+100$

$$\begin{array}{r} 101 \\ \times\, 50 \\ \hline \end{array}$$

ex: Find the indicated sum, if possible.

a) $a_n = 12n + 15, \quad S_{20} = ?$

$$S_{20} = \frac{20}{2}(27 + 255) = 2820$$

$n=20$

ex: Find the indicated sum, if possible.

b) $5 + 1 - 3 - 7 + \ldots \qquad S_{15} = ?$

$$S_{15} = \frac{15}{2}\left(5 + -51\right) = \boxed{-345}$$

$a_{15} = ?$

$a_n - 5 = -4(n-1)$

$a_n = -4n + 9$

$a_{15} = -4(15) + 9 = {}^{-}51$

$S_{15} = -345$

ex: Find the indicated sum, if possible.

c) $2 + 6 + 10 + \ldots + 58$

$S_n = \frac{n}{2}(2 + 58)$

write a rule

$a_n - 2 = 4(n-1)$

$a_n = 4n - 2$

$58 = 4n - 2$

$15 = n$

$S_{15} = \frac{15}{2}(60)$

$S_{15} = 450$

ex: Find the indicated sum, if possible.

d) $2 + 6 + 10 + 14 + \ ...$

*Not possible; infinite sum (arithmetic)*

Sum diverges

## Summation Notation

Summation Notation (a.k.a. Sigma Notation) is used to express a series.

Upper Limit (where to stop)

Sigma

$$\sum_{n=1}^{4} (3n+5)$$

Summand (Explicit Rule)

Lower Limit (where to begin)

ex: Find the sum, if possible.

a) $\sum_{n=1}^{15} 3n+5 = 435$

$S_{15} = \frac{15}{2}(8 + 50)$

b) $\sum_{i=0}^{\infty} i-2$ not possible sum diverges.

$$\sum_{n=1}^{4}(n^2+1) = 2+5+10+17$$

$$= 34$$

ex: Express the series using summation notation. Then find the sum or explain why there is no sum.

$a_n - 3 = -3(n-1)$

$a_n = -3n + 6$

a) 3, 0, -3, -6, -9

$$\sum_{n=1}^{5} (-3n+6) = -15$$

ex: Express the series using summation notation. Then find the sum or explain why there is no sum.

b) $1 + 2 + 3 + 4 + 5 \ldots$ $\infty$

$$\sum_{n=1}^{\infty} n$$

no sum
(infinite)
Sum diverges

ex: Express the series using summation notation. Then find the sum or explain why there is no sum.

c) 6 + 2 - 2 - 6 . . . - 78

rule: $a_n = -4n + 10$

$-78 = -4n + 10$

$22 = n$

$$\sum_{n=1}^{22} -4n + 10 = -792$$

$S_{22} = \frac{22}{2}(6 + -78)$