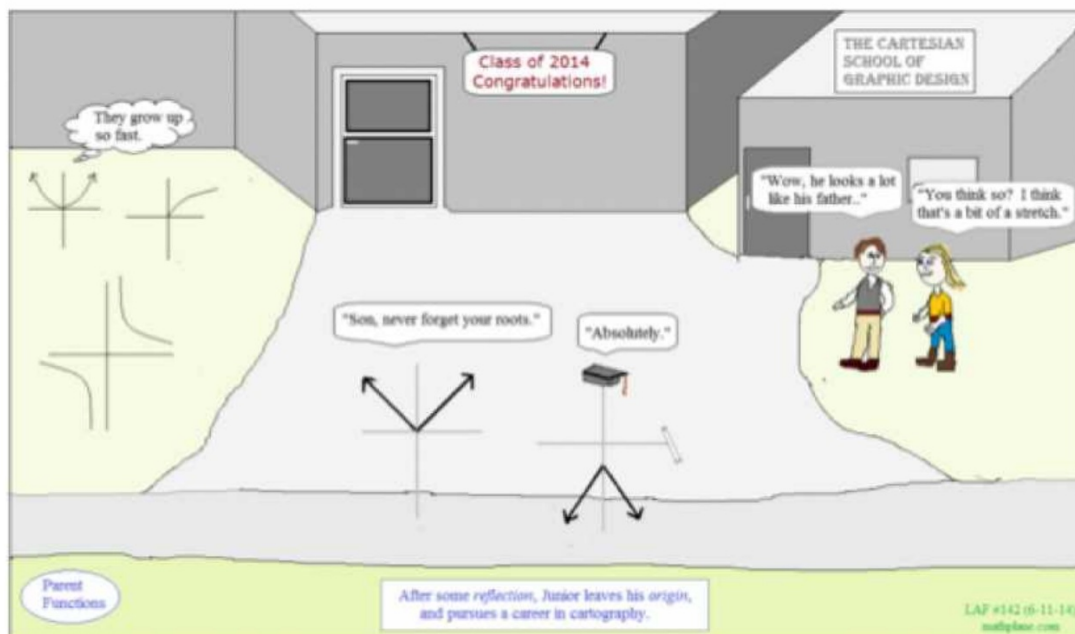


Tabular Data

5.7 Average Rate of Change



*See printout.

HW:

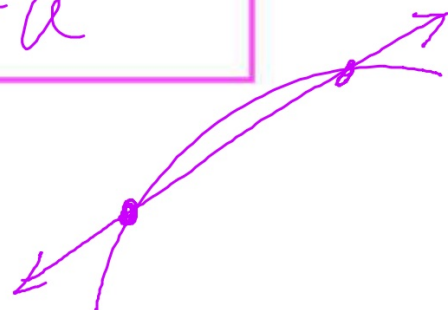
Average Rate of Change (ARC) (slope)

The average rate of change of the function $f(x)$ on the interval $x = a$ to $x = b$ is

$$\text{Average Rate of Change: } \frac{f(b) - f(a)}{b - a}$$

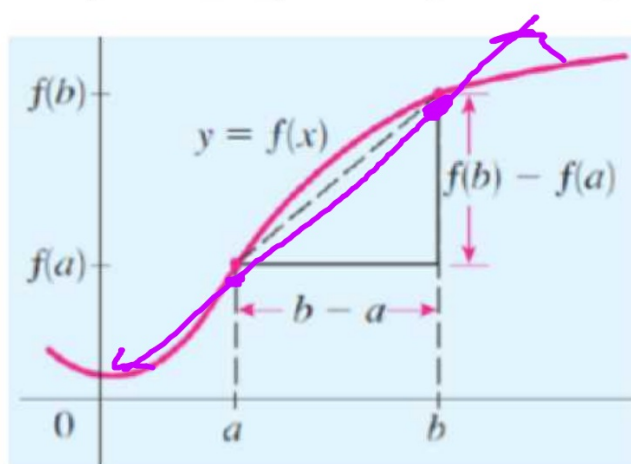
$$m = \frac{y_2 - y_1}{x_2 - x_1} =$$

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$



Graphical Perspective

The average rate of change of the function $f(x)$ on the interval $x = a$ to $x = b$ is the **slope** of the secant line between $x = a$ and $x = b$, that is, the line that passes through the points $(a, f(a))$ and $(b, f(b))$.



ex: Find the average rate of change over the indicated interval.

a) $f(x) = \frac{x-1}{x+2}$, $[a, b]$ $(0, -\frac{1}{2})$ $(3, \frac{2}{5})$
 $[0, 3]$

$$f(0) = -\frac{1}{2}$$

$$f(3) = \frac{2}{5}$$

$$\frac{f(b) - f(a)}{b - a}$$

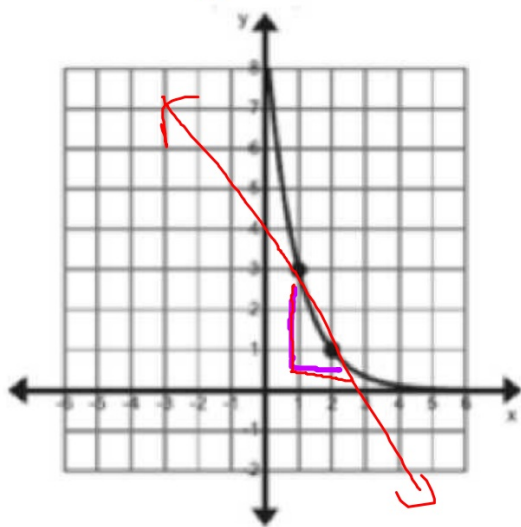
$$\frac{\frac{2}{5} - \left(-\frac{1}{2}\right)}{3 - 0} = \frac{10 \cdot \frac{2}{5} + \frac{1}{2} \cdot 10}{3 \cdot 10}$$

$$\frac{4 + 5}{30} = \frac{9}{30} = \frac{3}{10}$$

ex: Find the average rate of change over the indicated interval.

b) [1,2]

-2



$(1, 3)$ $(2, 1)$

$$\frac{1-3}{2-1} = -2$$

$$\frac{3-1}{1-2} = -2$$

ex: Find the average rate of change over the indicated interval.

c) $3 < t < 5$

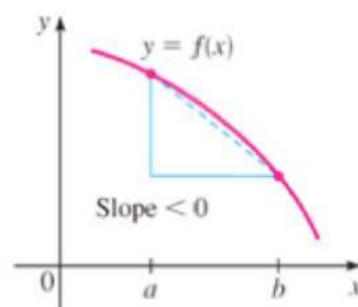
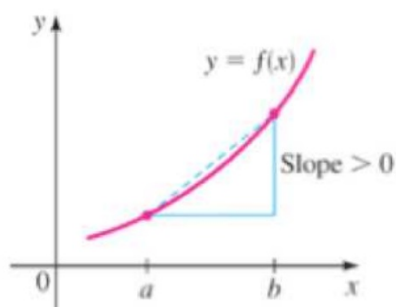
| | | | | | |
|--------------|----|----|----|----|----|
| Time (years) | 1 | 2 | 3 | 4 | 5 |
| Height(in.) | 27 | 35 | 37 | 42 | 45 |

$(3, 37)$ $(5, 45)$

$$\frac{45-37}{5-3} = 4 \text{ in/year}$$

- If $f(x)$ is strictly **increasing** on the interval $[a, b]$, then average rate of change of $f(x)$ is positive on the interval $[a, b]$.

- If $f(x)$ is strictly **decreasing** on the interval $[a, b]$, then average rate of change of $f(x)$ is negative on the interval $[a, b]$.



Tabular Data

1. Complete the following tables and answer the questions.

a)

| x | $y = 3x$ | 1 st Diff |
|----|----------|----------------------|
| -3 | -9 | } +3 } +3 } +3 |
| -2 | -6 | |
| -1 | -3 | |
| 0 | 0 | |
| 1 | 3 | |
| 2 | 6 | |
| 3 | 9 | |

This function is. linear quadratic exponential logarithmic

How do you know?

constant slope of 3

1. Complete the following tables and answer the questions.

b)

| x | $y = x^2$ | 1 st Diff | 2 nd Diff |
|-----|-----------|----------------------|----------------------|
| 0 | 0 | 1 3 5 7 | 2 2 2 |
| 1 | 1 | | |
| 2 | 4 | | |
| 3 | 9 | | |
| 4 | 16 | | |
| 5 | 25 | | |
| 6 | 36 | | |

This function is. linear quadratic exponential logarithmic

How do you know?

2nd differences are the same

1. Complete the following tables and answer the questions.

c)

| x | $y = 3^x$ |
|-----|-----------|
| 0 | 1 |
| 1 | 3 |
| 2 | 9 |
| 3 | 27 |
| 4 | 81 |
| 5 | 243 |
| 6 | 729 |

$$b=3$$

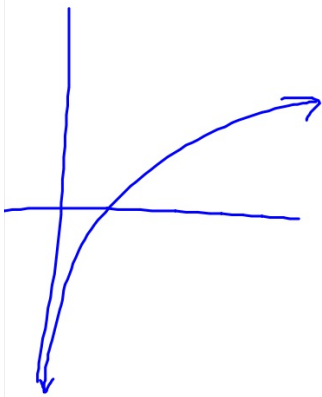
This function is. linear quadratic exponential logarithmic

How do you know?

*differences are increasing rapidly
multiplying by 3*

1. Complete the following tables and answer the questions.

d)



| x | $y = \log_3 x$ |
|-----|----------------|
| 1 | 0 |
| 3 | 1 |
| 9 | 2 |
| 27 | 3 |
| 81 | 4 |
| 243 | 5 |
| 729 | 6 |

This function is. linear quadratic exponential logarithmic

How do you know?

slow 'y' increase

difference of 1 on the 'y'

x-values are multiples of 3

2. Use differences to identify the type of function represented by the table of values. Then label which type of function each table of values models.

| | | | | | | | | | | |
|----|----|--|----|----|--|------|-----|--|----|----------------|
| x | y | | x | y | | x | y | | x | y |
| -4 | 5 | | -5 | 32 | | 0.5 | 0.9 | | -2 | $\frac{1}{16}$ |
| -3 | 8 | | -4 | 16 | | 0.75 | 1.1 | | -1 | $\frac{1}{4}$ |
| -2 | 13 | | -3 | 8 | | 1 | 1.3 | | 0 | 1 |
| -1 | 20 | | -2 | 4 | | 1.25 | 1.5 | | 1 | 4 |
| 0 | 29 | | -1 | 2 | | 1.5 | 1.7 | | 2 | 16 |
| 1 | 40 | | 0 | 1 | | 1.75 | 1.9 | | 3 | 64 |

Quadratic

exp.
(decay)

Linear

exp.
(growth)

3.

Rebecca records the amount of money, $f(x)$, in her bank account each month, x , as shown in the table.

Bank Account

| Month (x) | Amount of Money $f(x)$ |
|---------------|---------------------------|
| 0 | \$ 6 |
| 1 | \$12 |
| 2 | \$24 |
| 3 | \$48 |
| 4 | \$96 |

Create a function that models this relationship.

4.

A scientist studies several colonies of bacteria. She records the number of cells in the colony every hour. Several tables containing the data are shown.

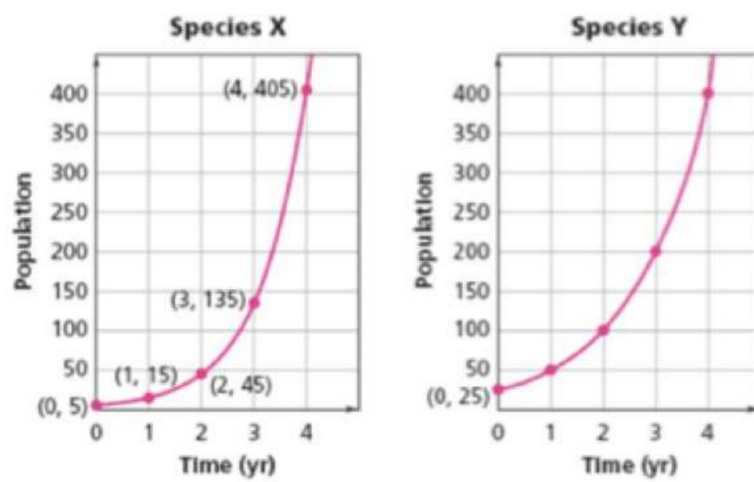
Click on the table or tables that represent exponential growth.

| Colony 1 | | Colony 2 | | Colony 3 | |
|----------|-------|----------|-------|----------|-------|
| Hours | Cells | Hours | Cells | Hours | Cells |
| 0 | 10 | 0 | 3 | 0 | 4.0 |
| 1 | 25 | 1 | 12 | 1 | 4.5 |
| 2 | 40 | 2 | 48 | 2 | 5.0 |
| 3 | 55 | 3 | 192 | 3 | 5.5 |

| Colony 4 | | Colony 5 | |
|----------|-------|----------|-------|
| Hours | Cells | Hours | Cells |
| 0 | 8 | 0 | 200 |
| 1 | 12 | 1 | 100 |
| 2 | 18 | 2 | 50 |
| 3 | 27 | 3 | 25 |

5a)

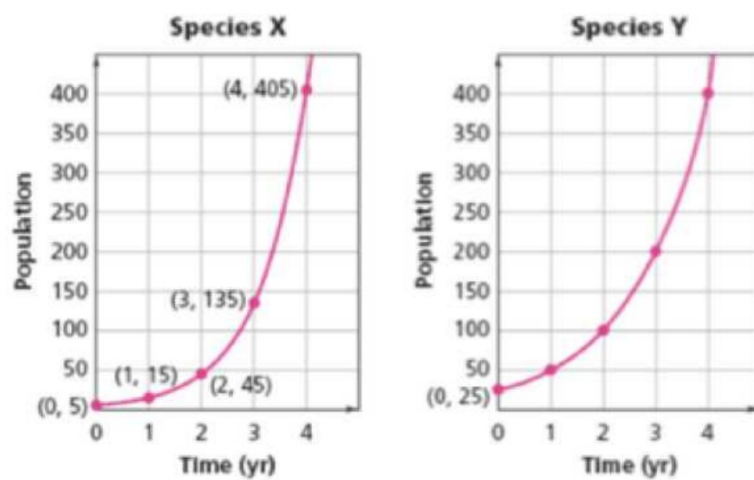
The following graphs show the population growth for two species.



Show both graphs are exponential. $y = ab^x$

5b)

The following graphs show the population growth for two species.



Write the exponential function $y = ab^x$ for species X and Y. What will be the population at time $t=6$ for both species?