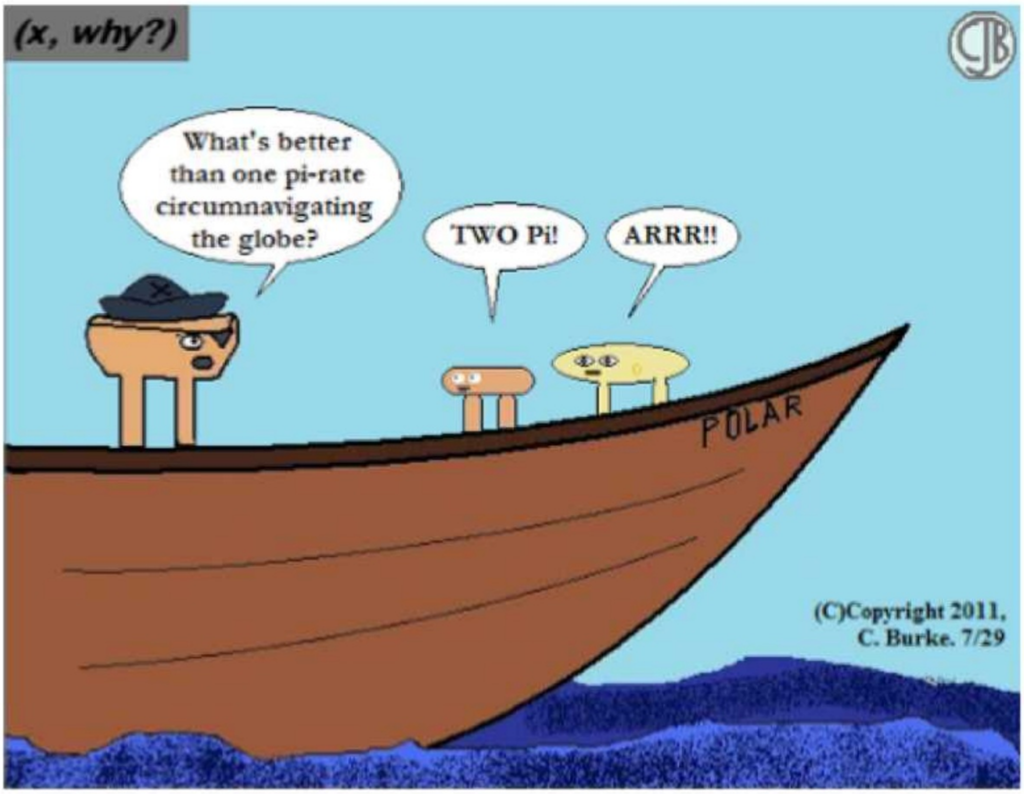
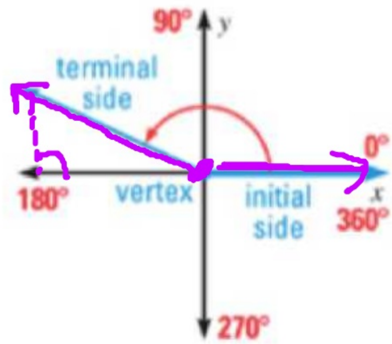
9.2 Degree and Radian Measure



Angles in Standard Position

In a coordinate plane, an angle can be formed by fixing one ray, called the **initial side**, and rotating the other ray, called the **terminal side**, about the vertex.

An angle is in **standard position** if its vertex is at the origin and its initial side lies on the positive x-axis.

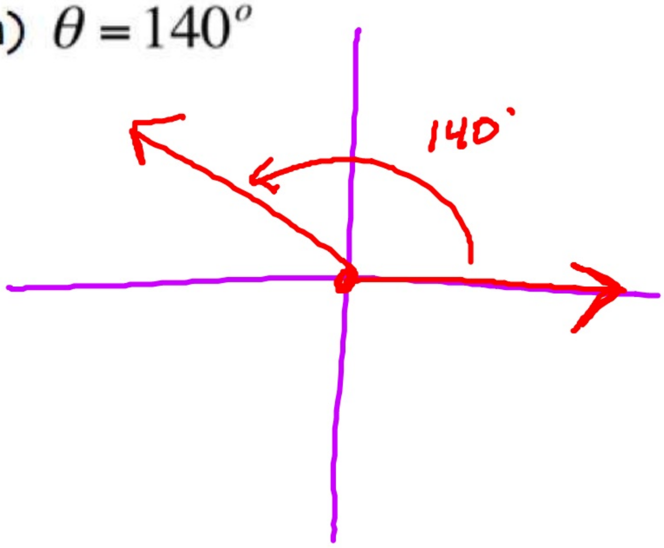


Rotations counter-clockwise: positive angles

Rotations clockwise: negative angles

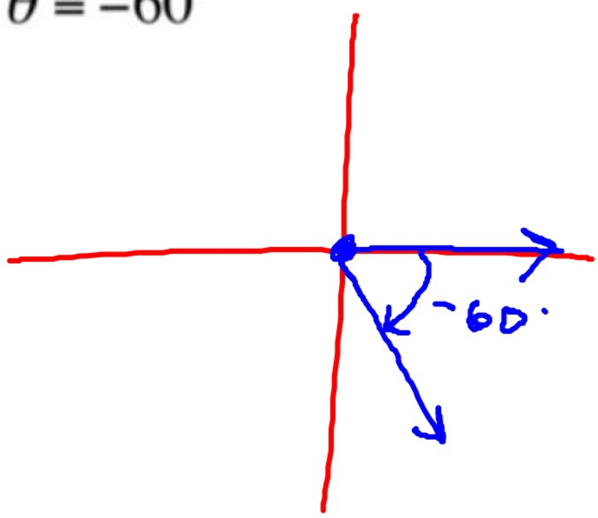
Example 1: Draw an angle with the given measure in standard position.

a) $\theta = 140^o$



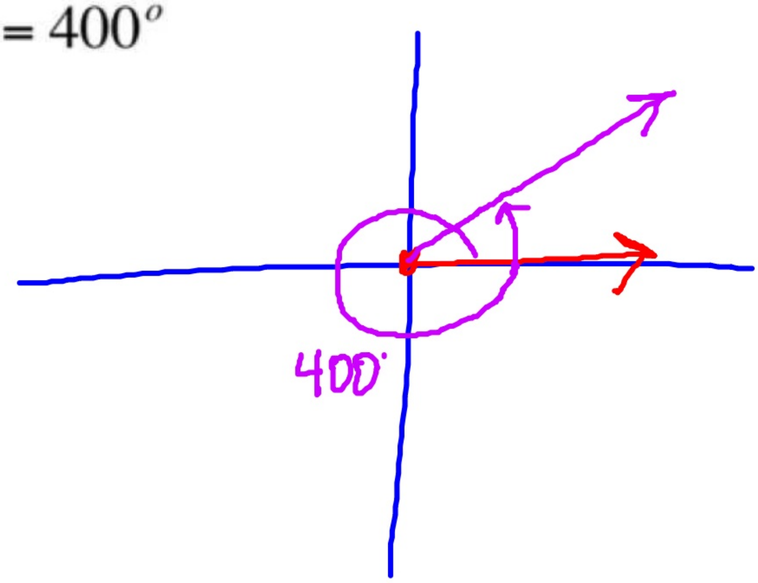
Example 1: Draw an angle with the given measure in standard position.

b) $\theta = -60^{o}$



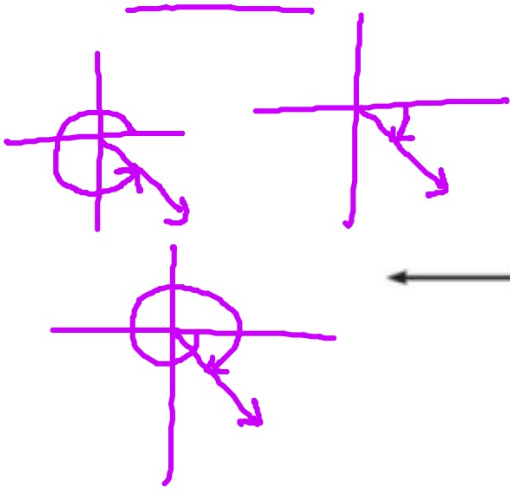
Example 1: Draw an angle with the given measure in standard position.

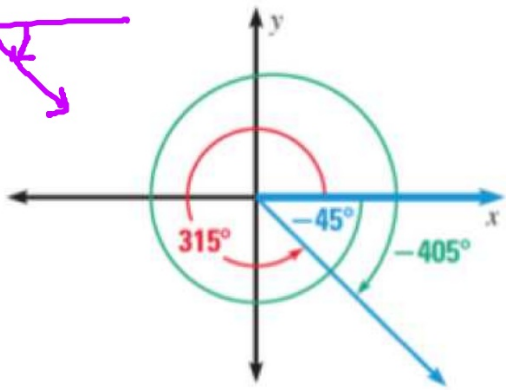
c) $\theta = 400^o$



Coterminal Angles -

Coterminal angles are angles whose terminal sides coincide.

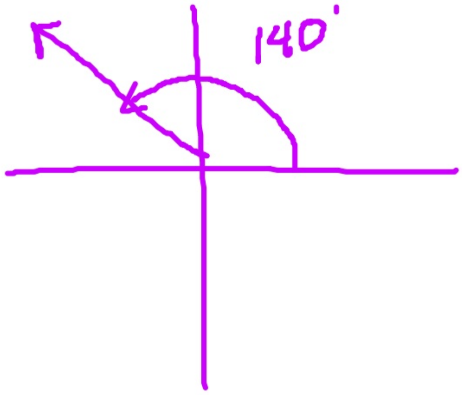




*To find a coterminal angle, add or subtract multiples of 360°.

Example 2: Find one positive and one negative coterminal angle for 140°.

a) $\theta = 140^o$



$\theta = 140^\circ - 360^\circ = -220^\circ$

$\theta = 140^\circ + 360^\circ = 500^\circ$

Example 2: Find one positive and one negative coterminal angle for 140°.

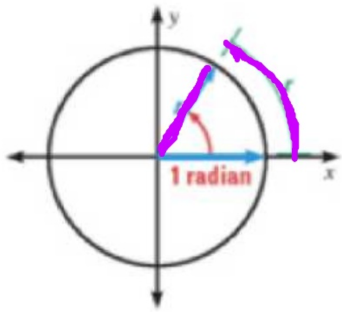
b) $\theta = -450^o$

neg.: -90° or -810° or...

pos: 270° or...

Radian Measure

Angles can also be measured in radians. One radian is the measure of an angle in standard position whose terminal side intercepts an arc of length r.



Because the circumference of a circle is $2\pi r$ there are 2π radians in a full circle.

Therefore:

$$360^o = 2\pi \text{ radians}$$

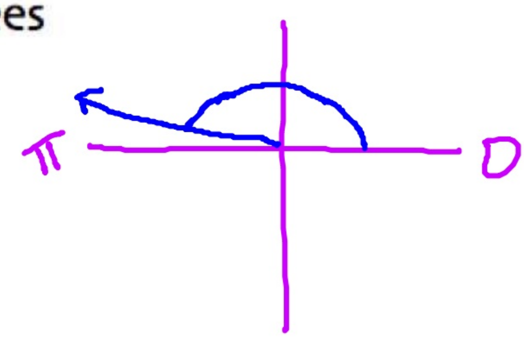
$$\boxed{180^o = \pi \text{ radians}}$$

Converting Angles - Degrees to Radians

multiply by $\frac{\pi}{180}$

Converting Angles - Radians to Degrees

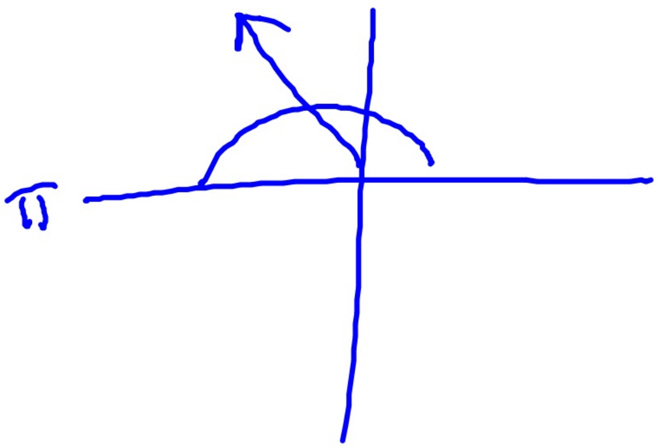
multiply by $\frac{180}{\pi}$



Ex 3: Convert from degrees to radians.

$$100^o$$

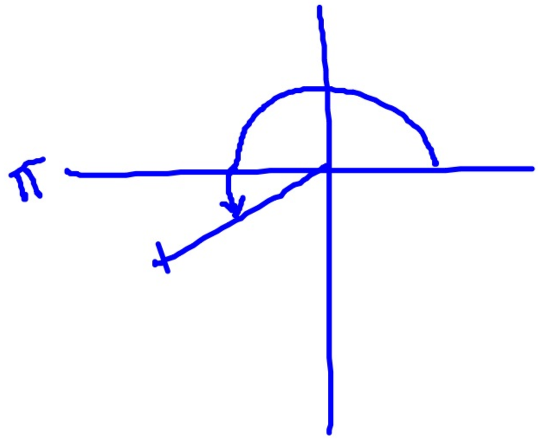
$$100^\circ \cdot \frac{\pi}{180} = \frac{10\pi}{18} = \frac{5\pi}{9}$$



Ex 4: Convert from radians to degrees.

$$\frac{7\pi}{6}$$

$$\frac{7\pi}{6} \cdot \frac{180}{\pi} = 210^{\circ}$$



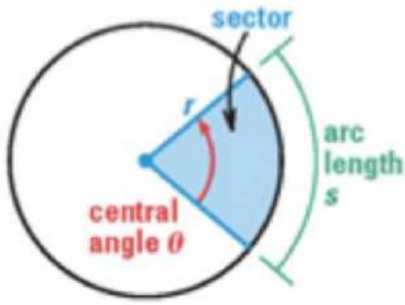
Arc Length and Area of a Sector

The arc length s and area A of a sector with radius r and central angle θ (measured in radians) are as follows.

Arc length: $s = r\theta$

Area: $A = \frac{1}{2}r^2\theta$

θ: radians



$s = r\theta$

Example 5: Find the arc lenath and area of a sector given r = 3 cm and central angle $\theta = 120^{o}$.

$120 \cdot \frac{\pi}{180} = \frac{2\pi}{3}$

a) arc length

$S = r\theta$

$S = (3)\left(\frac{2\pi}{3}\right)$

$S = 2\pi$ cm

Example 5: Find the arc lenath and area of a sector given r = 3 cm and central angle $\theta = 120^o$.

b) area

$$A = \frac{1}{2} r^2 \theta$$

$$A = \frac{1}{2}(3)^2 \frac{2\pi}{3}$$

$$A = 3\pi \text{ cm}^2$$

Example 6: Use a calculator to evaluate the trigonometric expression. Round to 3 decimal places.

a) $\cos\left(\frac{5\pi}{7}\right)$

$-.623$

b) $\cot(500^o)$

$\frac{1}{\tan 500^\circ} = -1.192$

c) $\csc\left(-300^o\right)$

$\frac{1}{\sin(-300^\circ)} = 1.154$

d) $\sin\left(\frac{11\pi}{5}\right) = .588$