19.) $2, \frac{3}{2}, \frac{9}{8}, \frac{27}{32}$

$a_1 = 2$

$r = \frac{3}{4}$

$a_n = 2\left(\frac{3}{4}\right)^{n-1}$

$a_7 = 2\left(\frac{3}{4}\right)^{6}$

$= \frac{2 \cdot 729}{4^6} = \frac{2 \cdot 729}{4096}$

43.) $a_2 \quad a_4$

$-40 \quad -10$

$-40 \cdot r^2 = -10$

$r^2 = \frac{1}{4}$

$r = \pm \frac{1}{2}$

$a_n = -80\left(\frac{1}{2}\right)^{n-1}$

53.) $\sum_{n=0}^{10} (-4)^n$

finite geo.

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

$$S_{11} = 1\left(\frac{1-(-4)^{11}}{1-(-4)}\right) = \frac{1+4^{11}}{5} = 838,861$$

17.) $\sum_{i=0}^{\infty} \left(\frac{-3}{7}\right)^i$

$$S = \frac{a_1}{1-r}$$

$$S = \frac{1}{1-\left(\frac{-3}{7}\right)} = \frac{1}{1+\frac{3}{7}} = \frac{7}{10}$$

4.) $a_0 = 4$

$a_n = 2a_{n-1}$

$a_1 = 2 \cdot 4 = 8$

$a_2 = 2 \cdot 8 = 16$

$a_3 = 2 \cdot 16 = 32$

$a_4 = 2 \cdot 32 = 64$

17) $S = \frac{a_1}{1-r} = \frac{1}{1-\left(-\frac{3}{7}\right)}$

$= \frac{1}{1+\frac{3}{7}}$

$|r|<1$

$-1<r<1$

$= \frac{1}{\frac{10}{7}}$

$= \frac{7}{10}$

11.) $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots + \frac{1}{6561}$

$a_n = 1\left(-\frac{1}{3}\right)^{n-1}$

$\frac{1}{6561} = \left(-\frac{1}{3}\right)^{n-1}$

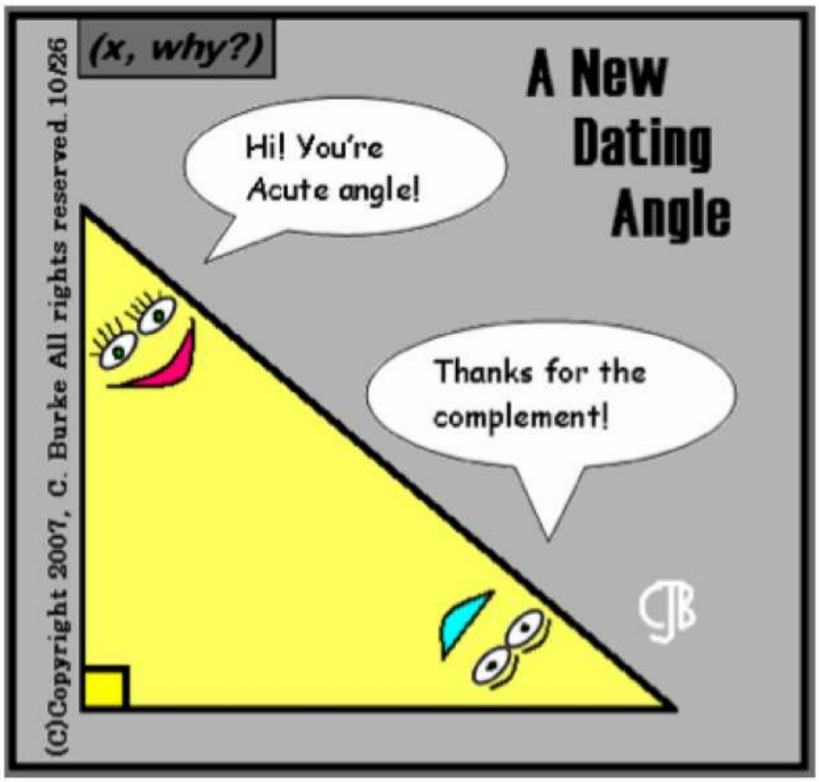
$\left(-\frac{1}{3}\right)^{8} = \left(-\frac{1}{3}\right)^{n-1}$

$9 = n$

$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$

$S_9 = 1\left(\frac{1 - \left(-\frac{1}{3}\right)^9}{1 - \left(-\frac{1}{3}\right)}\right)$

9.1 Right Triangle Trigonometry



What is trigonometry?

Trigonometry is is a branch of mathematics that studies relationships involving lengths and angles of triangles.

Commonly Used Greek Letters in Trigonometry Used to Represent Angle Measures:

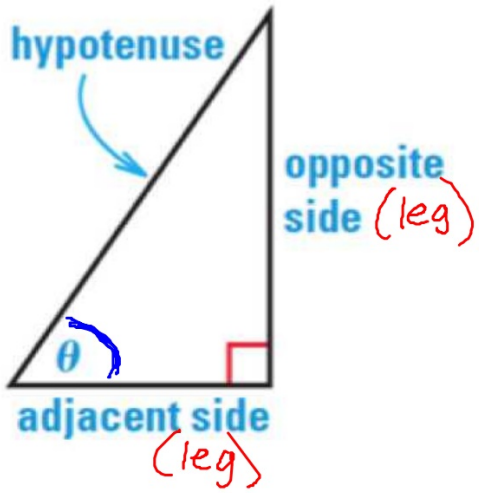
θ : theta

α : alpha

β : beta

SOHCAHTOA

SOH CAH TOA



$\sin\theta =$ opp. leg / hyp.

$\cos\theta =$ adj. leg / hyp

$\tan\theta =$ opp. leg / adj. leg

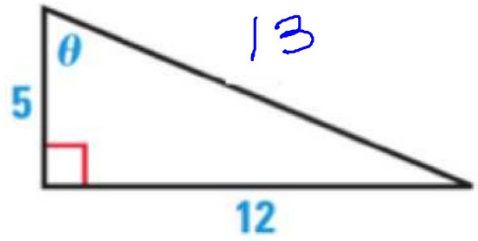
Reciprocal Trigonometric Ratios

Cosecant: $\csc\theta = \frac{1}{\sin\theta} =$ $\frac{\text{hyp.}}{\text{opp. leg}}$

Secant: $\sec\theta = \frac{1}{\cos\theta} =$ $\frac{\text{hyp.}}{\text{adj. leg}}$

Cotangent: $\cot\theta = \frac{1}{\tan\theta} =$ $\frac{\text{adj. leg}}{\text{opp leg}}$

ex: Find all trigonometric functions of theta.

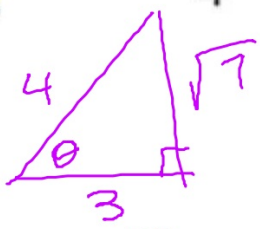


$\sin\theta = \frac{12}{13}$ $\quad \csc\theta = \frac{13}{12}$

$\cos\theta = \frac{5}{13}$ $\quad \sec\theta = \frac{13}{5}$

$\tan\theta = \frac{12}{5}$ $\quad \cot\theta = \frac{5}{12}$

ex: If $\cos\theta = \frac{3}{4}$ find the other trigonometric functions of theta.



$\sin\theta = \frac{\sqrt{7}}{4}$ $\quad \csc\theta = \frac{4}{\sqrt{7}} = \frac{4\sqrt{7}}{7}$

$\cos\theta = \frac{3}{4}$ $\quad \sec\theta = \frac{4}{3}$

$\tan\theta = \frac{\sqrt{7}}{3}$ $\quad \cot\theta = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$

Common Pythagorean Triplets

3 - 4 - 5

5 - 12 - 13

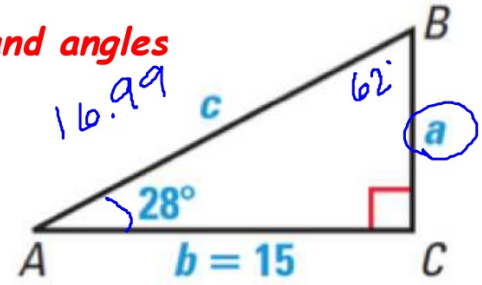
8 - 15 - 17

7 - 24 - 25

Memorize these!

ex: Solve ΔABC.

Find the missing sides and angles



$m\angle B = 62°$

$a = 7.97$

$c = 16.99$

Deg

find a

$\tan 28° = \frac{a}{15}$

$7.97 = a$

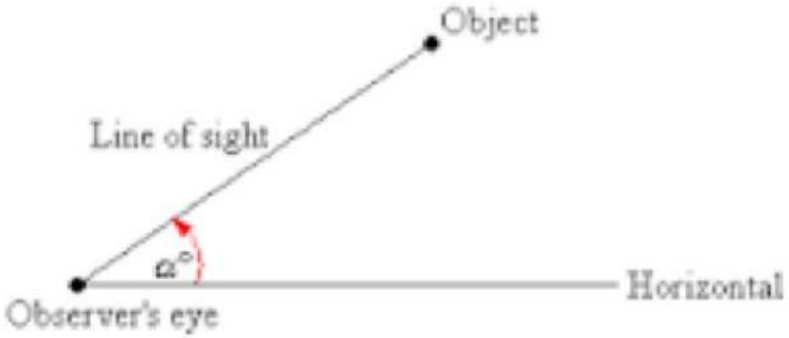
Find c

$\cos 28° = \frac{15}{c}$

$c = \frac{15}{\cos 28°} = 16.99$

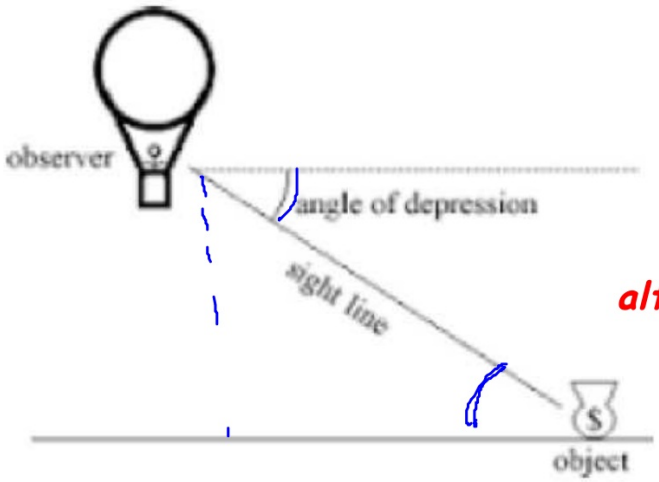
Vocabulary

- angle of elevation - the angle between one's line of sight and the horizontal

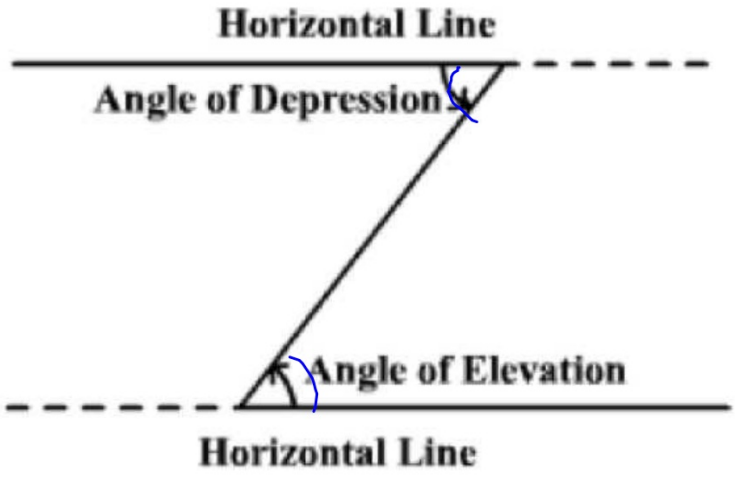


Vocabulary

- angle of depression - the angle between one's line of sight and the horizontal

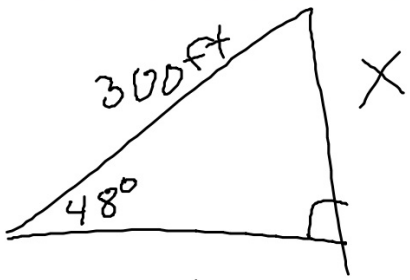


alternate interior



The angle of depression and elevation are CONGRUENT!

ex: A parasailer is attached to a boat with a rope 300 feet long. The angle of elevation from the boat to the parasailer is 48°. Estimate the parasailer's height above the boat.



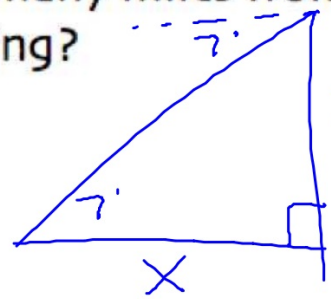
$\sin 48^\circ = \frac{X}{300}$

$222.9 ft = X$

ex: If a plane that is cruising at an altitude of 30,000 ft wants to land safely it must begin its descent so that the angle of depression to the airport is 7°.

1 mile = 5280 ft

a) How many miles from the airport must the plane begin descending?



$\tan 7° = \frac{30{,}000}{X}$

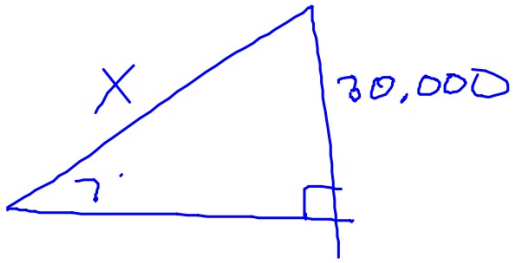
$X = 244330.329 \div 5280$

$X = 46.27$ miles

46.27

ex: If a plane that is cruising at an altitude of 30,000 ft wants to land safely it must begin its descent so that the angle of depression to the airport is 7°.

b) How many miles will the plane travel before landing?



$\sin 7^\circ = \frac{30000}{x}$

convert.

$x = 46.62$ miles

Ch 7 Review

ex: Determine whether the sequence is arithmetic, geometric, or neither. Then write an explicit and recursive rule.

2, -3, -8, -13 . . .

Explicit

$a_n - 2 = -5(n-1)$

$a_n = -5n + 7$

Recursive

$a_1 = 2$

$a_n = a_{n-1} - 5$

Ch 7 Review

ex: Use sigma notation to represent each series below. Then find the sum, if possible.

$a_n = a_1 \cdot r^{n-1}$

$a_n = 2 \cdot \left(\frac{1}{9}\right)^{n-1}$

infinte geometric

exists if $-1 < r < 1$

$$2 + \frac{2}{9} + \frac{2}{81} \ldots$$

$$\sum_{n=1}^{\infty} 2\left(\frac{1}{9}\right)^{n-1} = \frac{9}{4}$$

$$S = \frac{a_1}{1-r}$$

$$S = \frac{2}{1-\frac{1}{9}} = \frac{2}{\frac{8}{9}}$$

$$= \frac{9}{4}$$

Ch 7 Review

ex: Use sigma notation to represent each series below. Then find the sum, if possible.

geometric
finite

$5 + 10 + 20 + \ldots + 20480$

$$\sum_{n=1}^{13} 5(2)^{n-1}$$

$$S_{13} = 5\left(\frac{1-2^{13}}{1-2}\right) = 40955$$

$a_n = 5 \cdot 2^{n-1}$

$20480 = 5 \cdot 2^{n-1}$

$4096 = 2^{n-1}$

$2^{12} = 2^{n-1}$

$13 = n$

Ch 7 Review

ex: Find the indicated term.

Geometric Sequence, $a_3 = 12$, $a_6 = 96$, find a_{11}.

$12, \_\_, \_\_, 96$

$12r^3 = 96$

$r^3 = 8$

$r = 2$

$a_n = 3(2)^{n-1}$

$a_{11} = 3(2)^{10}$

$= 3072$

Ch 7 Review

ex: Find the sum of the first 203 positive odd integers.

$1 + 3 + 5 + \ldots\ldots 405$

$405 = a_{203}$

Arithmetic

$a_n = 2n - 1$

$a_{203} = 405$

$S_n = \frac{n(a_1 + a_n)}{2}$

$S_{203} = \frac{203(1 + 405)}{2}$

$= 41209$

Ch 7 Review

$-3, 2, 7, \ldots$

ex: How many terms of the arithmetic sequence must be added together for the sum of the series to be 116?

$d = 5$

$S_n = \frac{n(a_1 + a_n)}{2}$

Rule:

$a_n = 5n - 8$

$116 = \frac{n(-3 + 5n - 8)}{2}$

$232 = 5n^2 - 11n$

$0 = 5n^2 - 11n - 232$

$0 = (5n + 29)(n - 8)$

$n = \cancel{-\frac{29}{5}}, 8$

Ch 7 Review

ex: Generate the first 5 terms of the recursively-defined sequence.

$$a_1 = 3 \qquad a_2 = 2 \qquad a_n = a_{n-1} + (a_{n-2})^2 - 5$$

$a_3 = 2 + (3)^2 - 5 = 6$

$a_4 = 6 + (2)^2 - 5 = 5$

$a_5 = 5 + (6)^2 - 5 = 36$