55) $\sum_{i=1}^{n} (-5+7i) = 486$

$a_1 = 2$

$a_n = -5+7n$

$n = n$

$S_n = \frac{n(a_1+a_n)}{2}$

$486 = \frac{n(2+-5+7n)}{2}$

$972 = 7n^2 - 3n$

$0 = 7n^2 - 3n - 972$

$0 = (7n+81)(n-12)$

$\cancel{-\frac{81}{7}}, 12$

45.) $\sum_{i=5}^{14} (-54+9i)$

$a_{\cancel{\#}5} = -9$

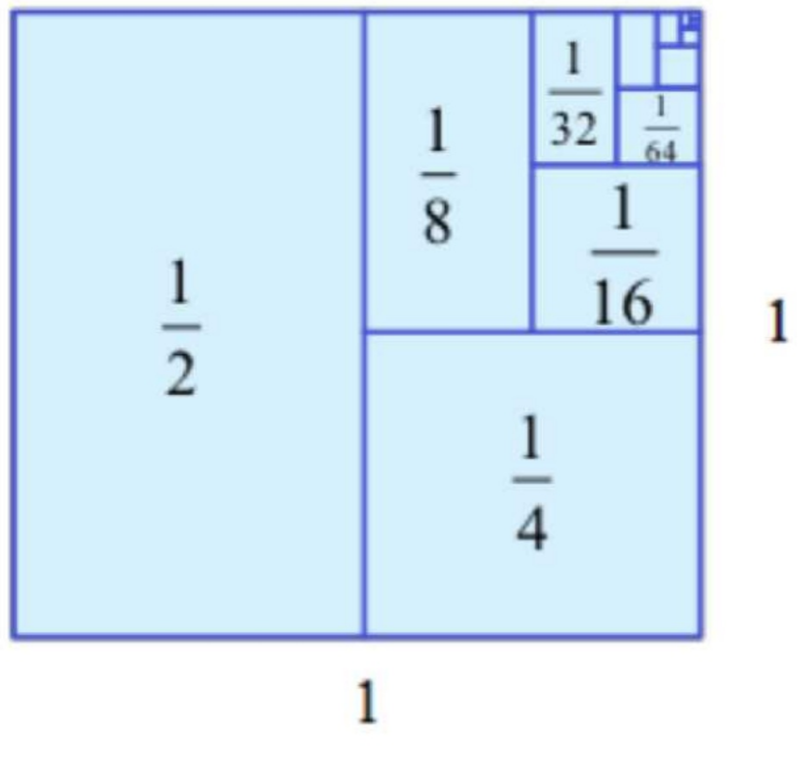
$a_{14} = 72$

$n = 10$

$S_{10} = \frac{10(-9+72)}{2}$

315

7.3/7.4 Geometric Sequences and Series



Geometric Sequences

In a **geometric sequence**, the ratio of any term to the previous term is constant. This constant ratio is called the **common ratio** and is denoted by *r*.

ex: Determine if the sequences is geometric. If so, identify the common ratio.

a) 10, 20, 30, 40, 50 . . .

No

b) 3, 6, 12, 24, 48 . . .

$r = 2$; Yes

$\frac{a_2}{a_1} = \frac{6}{3} = 2$

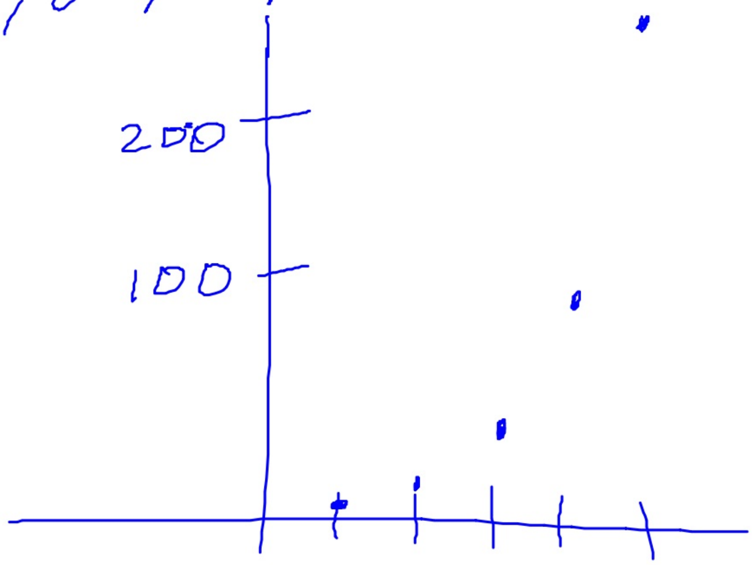
c) 16, 12, 9, 27/4 . . .

$r = \frac{3}{4}$; Yes

ex: Write the 1st 5 terms of the sequence and sketch the graph.

$$a_n = 3^n$$

3, 9, 27, 81, 243



Writing Explicit Rules for Geometric Sequences/Series

*Since geometric sequences have an exponential pattern, the explicit rule is exponential!

Recall Exponential Functions: $y = ab^x$

$\boxed{\text{Explicit Rule: } a_n = a_1 r^{n-1}}$

Where:

a_1 1st term

r common ratio

ex: Write an explicit rule for the geometric sequence.

a) 4, 20, 100, 500 . . .

$a_1 = 4$

$r = 5$

$a_n = 4(5)^{n-1}$

b) $a_2 = 3$, $r = 1/4$

$a_1 = 12$

$r = 1/4$

$a_n = 12\left(\frac{1}{4}\right)^{n-1}$

ex: Write an explicit rule for the geometric sequence.

c) $a_3 = 10, a_6 = 270$

$$a_n = \frac{10}{9}(3)^{n-1}$$

$$270 = a_1 r^5$$

$$270 = \frac{10}{r^2} \cdot r^5 \Longrightarrow 270 = 10r^3 ; r = 3$$

$$10 = a_1 r^2$$

$$\frac{10}{r^2} = a_1$$

$$10r^3 = 270$$

$$r = 3$$

Writing Recursive Rules for Geometric Sequences/Series

ex: Write a recursive rule for the geometric sequence.

a) 4, 20, 100, 500 . . .

previous term

$a_1 = 4$

$a_n = (a_{n-1}) \cdot 5$

b) $a_2 = 3$, $r = 1/4$

$$\boxed{\begin{aligned} a_1 &= 12 \\ a_n &= (a_{n-1})\tfrac{1}{4} \end{aligned}}$$

The Sum of a FINITE Geometric Sequence/Series

The Sum of a Finite Geometric Series

The sum of the first n terms of a geometric series with common ratio $r \neq 1$ is:

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$$

S_n	sum of the 1st n terms
n	number of terms in the sum
a_1	1st term in the sequence
r	common ratio

The Sum of an INFINITE Geometric Sequence/Series

The Sum of an Infinite Geometric Series

The sum of an infinite geometric series with first term a_1 and common ratio r is given by

$$S = \frac{a_1}{1-r}$$

provided $\boxed{|r| < 1}$. If $|r| \geq 1$, the series has no sum.

$-1 < r < 1$

S	sum of ALL terms
a_1	1^{st} term in the sequence/series
r	common ratio

ex: Find the indicated sum, if possible.

$a_1 = 1$

$r = 2$

$n = 9$

a) 1, 2, 4, 8, ... $\quad S_9 = ?$

finite geometric $(r = 2)$

$$S_9 = 1\left(\frac{1-2^9}{1-2}\right) = \frac{1-512}{-1} = 511$$

ex: Find the indicated sum, if possible.

b) $\sum_{n=1}^{8} 6\left(-\frac{1}{2}\right)^{n-1}$

$a_1 = 6$

$r = -\frac{1}{2}$

$n = 8$

$$S_8 = 6\left(\frac{1-\left(-\frac{1}{2}\right)^8}{1-\left(-\frac{1}{2}\right)}\right) = 6\left(\frac{1-\frac{1}{256}}{\frac{3}{2}}\right)$$

$$= 6\left(\frac{\frac{255}{256}}{\frac{3}{2}}\right) = \overset{1}{\cancel{6}}\,\overset{1}{\cancel{2}}\left(\frac{255}{\underset{\underset{64}{\cancel{128}}}{\cancel{256}}} \cdot \frac{\overset{1}{\cancel{2}}}{\cancel{3}}\right) = \frac{255}{64}$$

ex: Find the indicated sum, if possible.

c) $\sum_{n=1}^{\infty} 6\left(-\frac{1}{2}\right)^{n-1}$

$$S = \frac{6}{1-\left(-\frac{1}{2}\right)} = 4$$

$a_1 = 6$

$r = \frac{-1}{2}$

$$S_5 = 6 - 3 + \frac{6}{4} - \frac{6}{8} + \frac{6}{16} = 4.125$$

ex: Find the indicated sum, if possible.

d) 4 - 2 + 1 - 0.5 + ...

infinite geo.

$r = -\frac{1}{2}$

$a_1 = 4$

$|r| < 1$

$S = \frac{4}{1-(-\frac{1}{2})} = \frac{8}{3}$

ex: Find the indicated sum, if possible.

e) $9 + 6 + 3 + 0 - 3 - \ldots$

infinite arithmetic $(d = -3)$

No sum

ex: Find the indicated sum, if possible.

f) $\sum_{n=0}^{6} n^2 = 0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2$

$= 91$

ex: Express the series using summation notation. Then find the sum or explain why there is no sum.

a) 5 + 15 + 45 + 135 + . . .

$|r| > 1 \quad r = 3$

$$\sum_{n=1}^{\infty} 5(3)^{n-1}$$

no sum

ex: Express the series using summation notation. Then find the sum or explain why there is no sum.

b) 5 + 15 + 45 + 135

$$\sum_{n=1}^{4} 5(3)^{n-1} = 200$$

$$\sum_{n=1}^{4} 5 \cdot 3^n \cdot (3^{-1}) = \sum_{n=1}^{4} \frac{5}{3} \cdot 3^n$$

equivalent

ex: Express the series using summation notation. Then find the sum or explain why there is no sum.

c) 100 + 20 + 4 + 4/5 + . . .

$$\sum_{n=1}^{\infty} 100\left(\frac{1}{5}\right)^{n-1}$$

$$S = \frac{100}{1-\frac{1}{5}}$$

$$= 125$$

ex: Solve for x.

a) $\sum_{i=1}^{x} (5 - 5i) = -50$

$\boxed{x = 5}$

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$-50 = \frac{x(0 + 5 - 5x)}{2}$$

$$-100 = 5x - 5x^2$$

$$5x^2 - 5x - 100 = 0$$

$$5(x^2 - x - 20) = 0$$

$$5(x-5)(x+4) = 0$$

$$x = 5, \cancel{-4}$$

ex: Solve for x.

b) $\sum_{n=0}^{\infty} 3\left(\frac{x}{2}\right)^n = 7$

$X = \frac{8}{7}$

$S = \frac{a_1}{1-r}$

$7 = \frac{3}{1-\frac{x}{2}}$

$7\left(1-\frac{x}{2}\right) = 3$

$7 - \frac{7x}{2} = 3$

$+\frac{7x}{2} = +4$

$x = \frac{8}{7}$

ex: Find the explicit rule for. . .

$$\log x,\ \log\sqrt{x},\ \log\sqrt[4]{x}\ \ldots$$

$$\log x,\ \frac{1}{2}\log x,\ \frac{1}{4}\log x, \ldots$$

$$a_n = \left(\frac{1}{2}\right)^{n-1}\log x = \log x^{\left(\frac{1}{2}\right)^{n-1}}$$

ex: Find the sum of the first 15 three digit whole numbers ending in 5.

$105 + 115 + 125 + \ldots + \underline{245}$

write the nth

$a_n = 10n + 95$

$$S_n = \frac{n(a_1 + a_n)}{2}$$

2625

ex: Find the missing terms of the arithmetic sequence.

. . . 37, 33, 29, 25 . . .

?

$$\begin{array}{r} 33 \\ +25 \\ \hline 58/2 \end{array}$$

ex: Find the missing terms of the geometric sequence.

$$\ldots 48, \underline{\tfrac{48}{3}}, \underline{\tfrac{48}{9}}, \cancel{48/9}\ \ldots$$

48/27

$$48 \cdot r^3 = \frac{48}{27}$$

$$r^3 = \frac{1}{27}$$

$$r = \frac{1}{3}$$