13. $$\frac{LCM\cdot\frac{x-6}{6}-\frac{x-2}{x-6}\cdot LCM}{LCM\cdot\frac{36}{x-2}+\frac{4}{9}\cdot LCM}$$

LCM

$18(x-6)(x-2)$

$$\frac{3(x-6)^2(x-2)-\left[18(x-2)^2\right]}{18\cdot 36(x-6)+4\cdot 2(x-6)(x-2)}$$

648

35.) $$\frac{LCM\cdot\frac{3}{x-2} - \frac{6}{(x-2)(x+2)}\cdot LCM}{LCM\cdot\frac{3}{x+2} + \frac{1}{x-2}\cdot LCM}$$

LCM
$(x-2)(x+2)$

$$\frac{3(x+2) - [\,6\,]}{3(x-2) + x+2} = \frac{3x}{4x-4}$$

$$\frac{3x}{4(x-1)}$$

38.) $$\frac{-(x^3-x-1)}{5(x+1)}$$

25.) $$\frac{-2(2x^2+3x+3)}{(x-3)(x+3)(x+1)}$$

38.) $$\frac{\frac{1}{x} - \left(\frac{x}{\frac{1}{x}+1}\right)}{\frac{5}{x}}$$

$$\frac{x\cdot x}{x\cdot\frac{1}{x}+1\cdot x} = \frac{x^2}{x+1}$$

$$\frac{\frac{1}{x} - \frac{x^2}{x+1}}{\frac{5}{x}} = \left(\frac{1\cdot x+1}{x} - \frac{x^2\cdot x}{x+1}\right) \div \frac{5}{x}$$

$$\left(\frac{x+1-x^3}{\cancel{x}(x+1)}\right)\cdot\frac{\cancel{x}}{5}$$

$$\frac{(-x^3+x+1)}{5(x+1)}$$

23) $\frac{x \cdot 5}{x \cdot 6(x+3)} + \frac{(x+4)3(x+3)}{2x \cdot 3(x+3)}$   LCM $6(x+3)x$

$$\frac{5x + 3(x^2+7x+12)}{6x(x+3)}$$

$$\frac{3x^2+26x+36}{6x(x+3)}$$

$$\frac{x+7+1}{x+1}$$

$$\frac{(x+8)}{(x+1)}$$

## 5.6 Solve Rational Equations
## 5.7 Average Rate of Change

ex: Solve.

a) $$\frac{4}{x+3}+\frac{5}{6}=\frac{23}{18}$$

$$\frac{18\cdot 4}{18\cdot x+3}+\frac{5\cdot 3(x+3)}{6\cdot 3(x+3)}-\frac{23(x+3)}{18(x+3)}=0$$

$$\frac{72+15(x+3)-23(x+3)}{18(x+3)}=0$$

$$\frac{48-8x}{18(x+3)}=0$$

***Steps for solving:***

***1) Set the equation = 0***
***2) Find the LCD; add the fractions and simplify***
***3) Set the numerator = 0 (these are solutions)***
***4) Set the denominator = 0 (these are undefined; not solutions)***

LCD: $18(x+3)$

| Num | Den. |
|---|---|
| $48-8x=0$ | $18(x+3)=0$ |
| $x=6$ | $x=-3$ where the equation is undefined |

ex: Solve.

b) $\dfrac{2x}{x+5} - \dfrac{x^2-x-10}{x^2+8x+15} = \dfrac{3}{x+3}$

$(x+5)(x+3)$

$$\frac{(x+3)\,2x}{(x+3)\,x+5} - \frac{x^2-x-10}{(x+5)(x+3)} - \frac{3\cdot(x+5)}{x+3\,(x+5)} = 0$$

LCD: $(x+5)(x+3)$

$$\frac{2x(x+3) - [x^2-x-10] - [3(x+5)]}{(x+5)(x+3)} = 0$$

$$\frac{2x^2+6x-x^2+x+10-3x-15}{(x+5)(x+3)} = 0$$

$$\frac{x^2+4x-5}{(x+5)(x+3)} = 0$$

$$\frac{\cancel{(x+5)}(x-1)}{\cancel{(x+5)}(x+3)} = 0$$

| Num | Den. |
|---|---|
| $=0$ | $=0$ |
| ~~$x=-5$~~ | $x=-5$ |
| $1=x$ | $-3$ |

ex: Solve.

c) $1-\dfrac{8}{x-5}=\dfrac{3}{x}$

LCD: $x(x-5)$

$$1-\frac{8}{x-5}-\frac{3}{x}=0$$

$$\frac{x(x-5)-[8x]-[3(x-5)]}{x(x-5)}=0$$

$$\frac{x^2-16x+15}{x(x-5)}=0$$

| Num | Den |
|---|---|
| $x=1$ | $x=0$ |
| $15$ | $5$ |

ex: Solve.

d) $\dfrac{2}{x+1} - \dfrac{1}{x-1} = \dfrac{-2}{x^2-1}$

$\dfrac{(x-1)\,2}{(x-1)\,x+1} - \dfrac{1\,(x+1)}{x-1\,(x+1)} + \dfrac{2}{(x+1)(x-1)} = 0$

$\dfrac{2x-2-[x+1]+2}{(x+1)(x-1)} = 0$

$\dfrac{x-1}{(x+1)(x-1)} = 0$

$\cancel{x=1}$

No solution

ex: Solve.

e) $x - \dfrac{5}{x+1} = 2$

Average Rate of Change (slope)

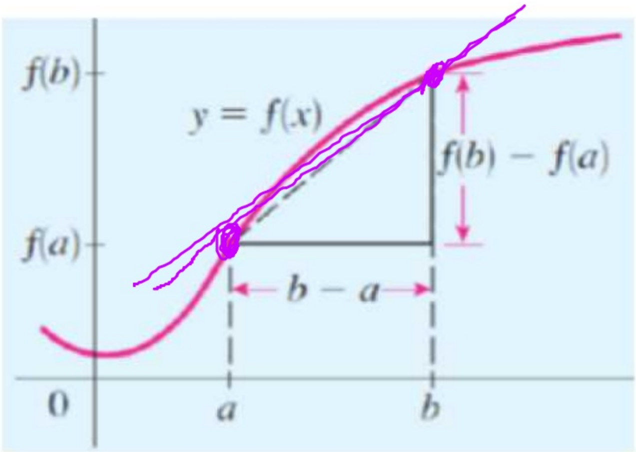
The average rate of change of the function $f(x)$ on the interval $x = a$ to $x = b$ is

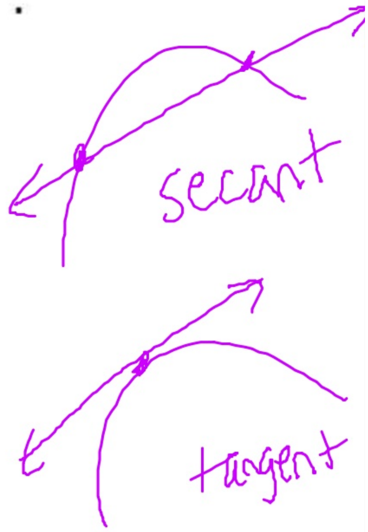
Average Rate of Change: $\boxed{\dfrac{f(b) - f(a)}{b - a}}$

*See printout.

## Graphical Perspective

The average rate of change of the function $f(x)$ on the interval $x = a$ to $x = b$ is the slope of the secant line between $x = a$ and $x = b$, that is, the line that passes through the points $(a, f(a))$ and $(b, f(b))$.



ex: Find the average rate of change over the indicated interval.

"x"

$(0, -\frac{1}{2})(3, \frac{2}{5})$

a) $f(x)=\frac{x-1}{x+2}$, [0,3]

[a, b]

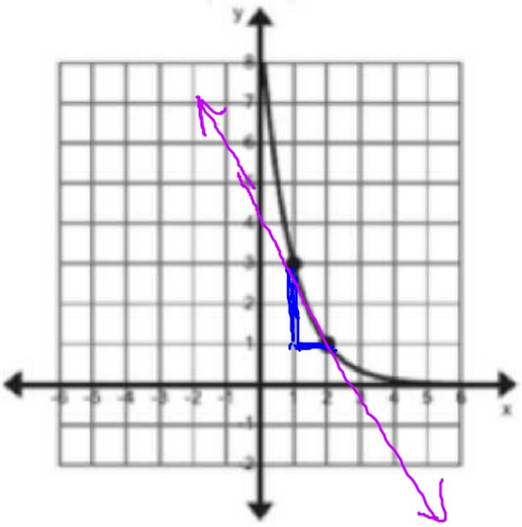
$$\frac{\frac{2}{5}-\left(\frac{-1}{2}\right)}{3-0} = \frac{10\cdot\frac{2}{5}+\frac{1}{2}\cdot 10}{10\cdot\frac{3}{1}} = \frac{9}{30}$$

$$\frac{3}{10}$$

ex: Find the average rate of change over the indicated interval.

b) [1,2]

-2



ex: Find the average rate of change over the indicated interval.

c) 3<t<5

| Time (years) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Height(in.) | 27 | 35 | 37 | 42 | 45 |

(3, 37), (5, 45)

$\boxed{4\ \text{in/year}}$

ex: On which interval does the function $f(x) = \frac{1}{8}x^3 - x^2$ have an average rate of change of 0.5?

| |
|---|
| $-2 < x < 2$ |
| $0 < x < 4$ |
| $-3 < x < 2$ |
| $-4 < x < 1$ |

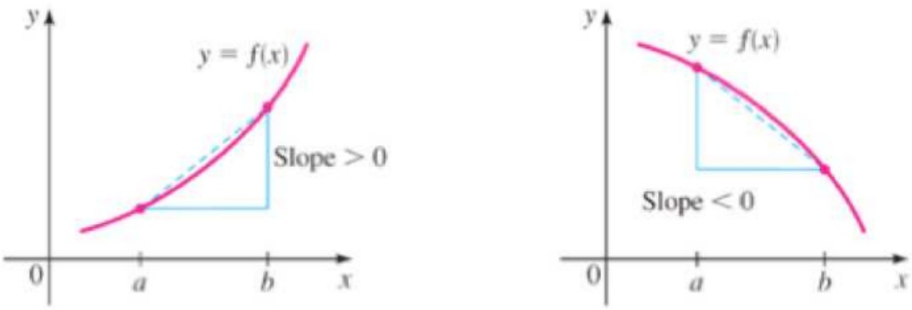
$(-2, -5)\ (2, -3)$

$$\frac{-3-(-5)}{2-(-2)} = \frac{2}{4}$$

$$\frac{1}{2}$$

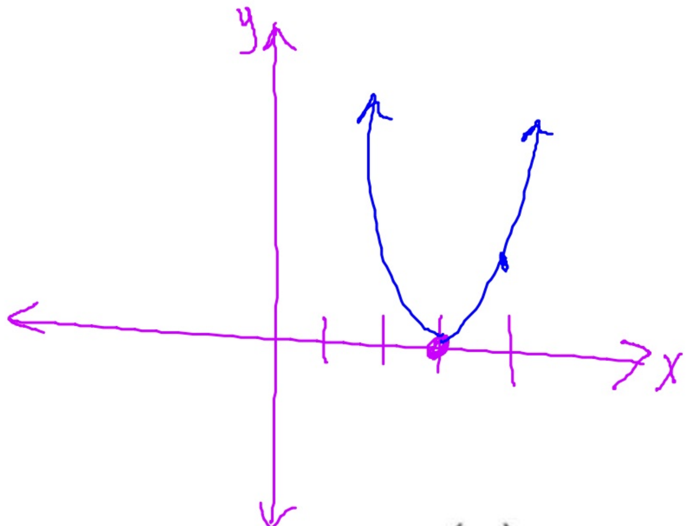
- If $f(x)$ is strictly increasing on the interval [a, b], then average rate of change of $f(x)$ is
  ______positive______on the interval [a, b].

- If $f(x)$ is strictly decreasing on the interval [a, b], then average rate of change of $f(x)$ is
  ______negative______on the interval [a, b].



ex:

a) Sketch: $f(x) = (x-3)^2$



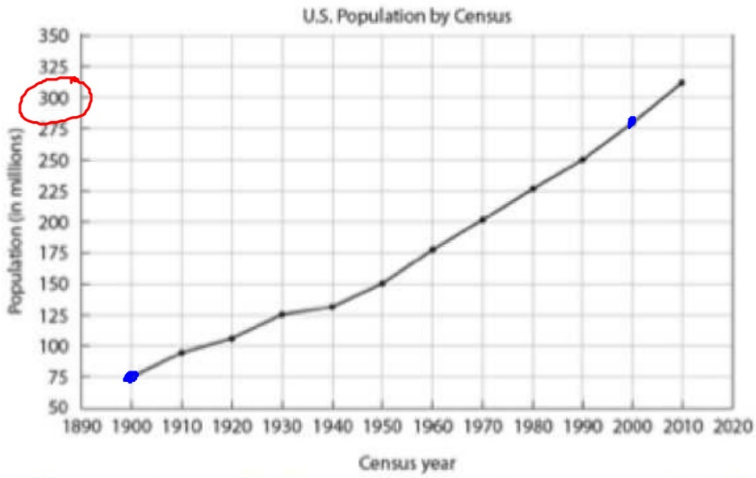

b) State an interval on which the average rate of $f(x)$ is positive. How do you know?

$[3,4]$

c) State an interval on which the average rate of $f(x)$ is negative. How do you know?

$[0,2]$

ex: The graph below shows the United States population from 1900 to 2010, as recorded by the U.S. Census Bureau.

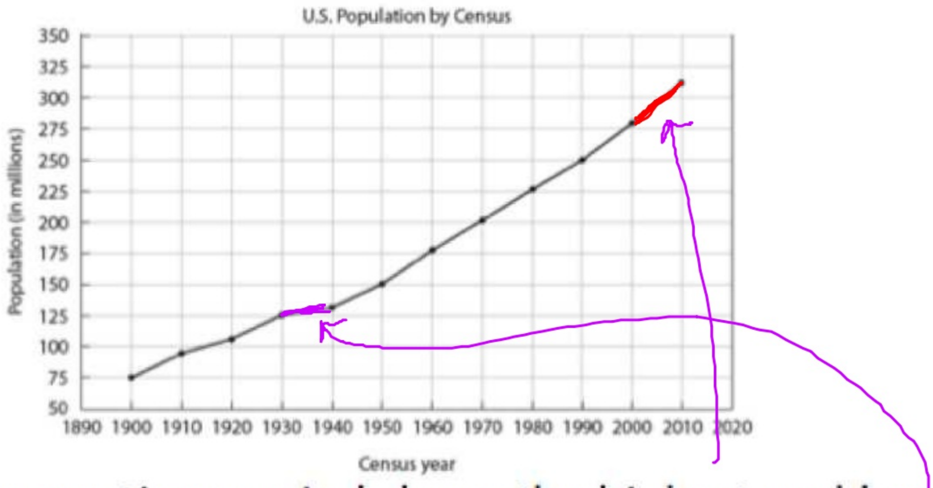



a) What was the rate of change in the populationfrom 1900 to 2000? Is this greater or less than the rate of change in the population from 2000 to 2010?

(1900, 75) (2000, 280) ; $\frac{41}{20}$

(2000, 280) (2010, 315) ; $\frac{35}{10} = \frac{7}{2}$ *

ex: The graph below shows the United States population from 1900 to 2010, as recorded by the U.S. Census Bureau.




b) Which 10-year time periods have the highest and lowest rates of change? How did you know?

*Add/Subtract/Multiply/Divide rational expressions*

*Simplify complex fractions*

*Solve rational equations (check for extraneous)*

*Average rate of change (slope)*