

$$17) \frac{1}{2x} + \frac{3}{x+7} + \frac{1}{x} = 0 \quad 2x(x+7)$$

$$\frac{1 \cdot (x+7) + 3 \cdot 2x + 1 \cdot 2(x+7)}{2x(x+7)} = 0$$

$$\frac{x+7 + 6x + 2x + 14}{2x(x+7)}$$

$$\left(-\frac{7}{3} \right)$$

$$\frac{9x+21}{2x(x+7)} = 0$$

$$\frac{1}{4} + \frac{1}{2}$$
$$\frac{1}{2 \cdot 2} + \frac{1}{2}$$

$$\frac{3}{x} + \frac{2}{x}$$

$$22.) \frac{6x(x-1)}{x+4} + \frac{4(x^2+3x-4)}{1} - \frac{(2x+2)(x+4)}{x-1} = 0$$

$$0 = \frac{6x^2 - 6x + 4x^2 + 12x - 16 - (2x^2 + 10x + 8)}{(x+4)(x-1)}$$

$$0 = \frac{8x^2 - 4x - 24}{(x+4)(x-1)} = \frac{4(2x^2 - x - 6)}{(x+4)(x-1)} = \frac{4(2x+3)(x-2)}{(x+4)(x-1)}$$

$\frac{-3}{2}, 2$

$$19.) \frac{5}{(x+3)(x-2)} - \frac{2}{1} - \frac{(x-3)(x+3)}{x-2} = 0$$

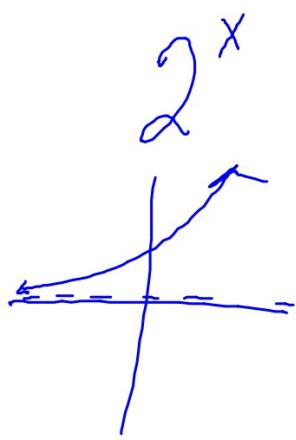
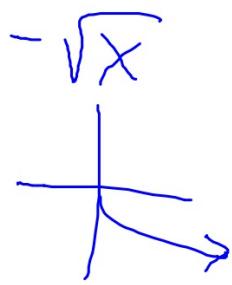
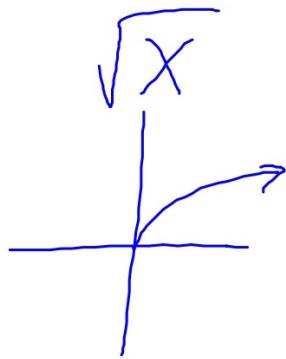
$$\frac{5 - 2(x^2 + x - 6) - (x^2 - 9)}{(x+3)(x-2)} = 0$$

$$\frac{-3x^2 + 2x + 26}{(x+3)(x-2)} = 0$$

Quad.
Formel

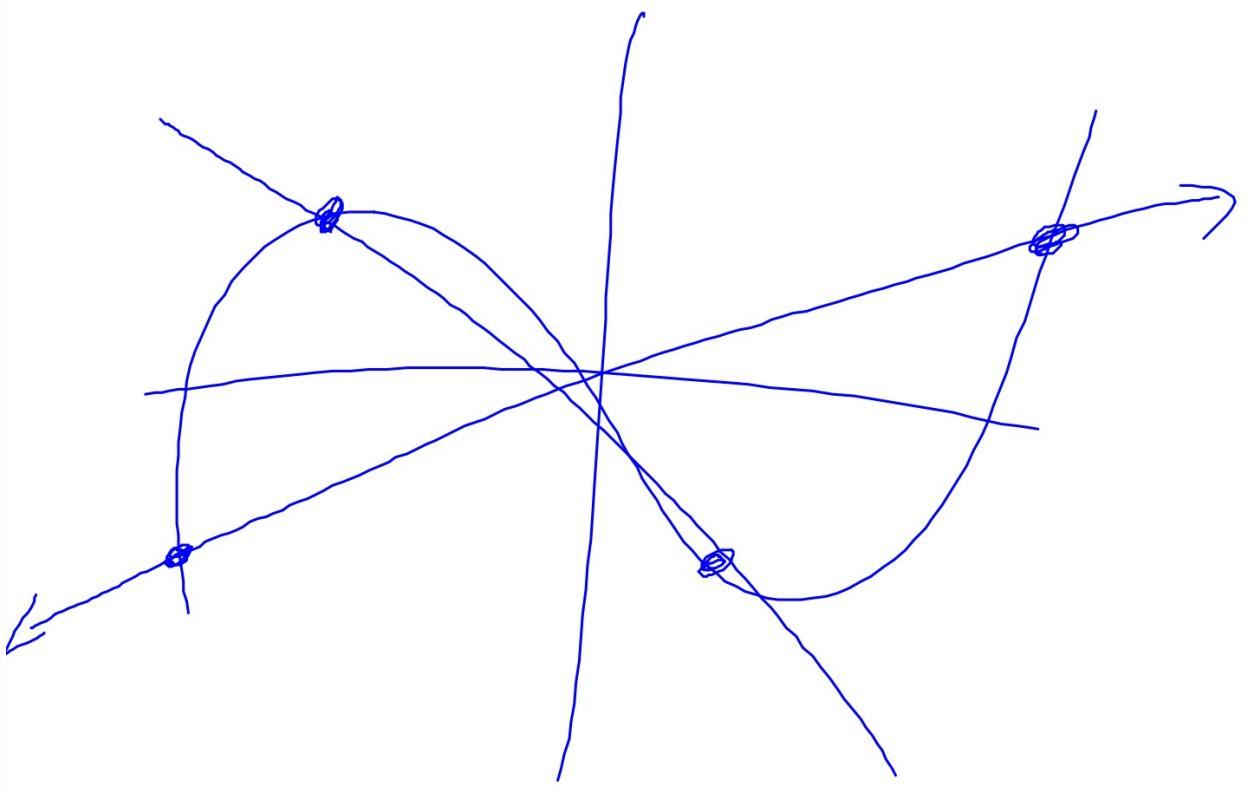
—

$$x = \frac{-2 \pm \sqrt{4 - 4(3)(-26)}}{2(3)}$$



$$\left| \begin{array}{l} \text{a.) } \boxed{-6} = \boxed{-6} \\ \text{d.) } \frac{m(0)-m(-3)}{0-(-3)} \\ \qquad \qquad \qquad \frac{3-(-6)}{3} > \end{array} \right| \quad \frac{2}{3}$$

The left side of the vertical bar contains two equations: a) $\boxed{-6} = \boxed{-6}$ and d) $\frac{m(0)-m(-3)}{0-(-3)}$. The right side of the vertical bar shows the fraction $\frac{3-(-6)}{3} >$, which simplifies to $\frac{9}{3} >$.



$$18) \frac{1}{x-2} + \frac{2}{x+2} = 0$$

$$\frac{x^2 + 2x - 8 - 3x^2 + 6x}{(x-2)(x+2)} = 0$$

$$\frac{-x^2 + 7x - 6}{(x-2)(x+2)} = 0$$

$$\frac{-(x^2 - 7x + 6)}{(x-2)(x+2)} = 0$$

$$\frac{-(x-6)(x-1)}{(x-2)(x+2)}$$

$$x=1, 6$$

18. 1, 6

20. -2, 3

22. $-\frac{3}{2}$, 2

24. \emptyset

$$19) \quad \frac{5}{x^2+x-6} - \frac{2}{1} - \frac{x-3(x+3)}{x-2(x+3)} = 0$$

$$\frac{5-2(x^2+x-6)-(x^2-9)}{(x+3)(x-2)} = 0$$

$$\frac{-3x^2-2x+26}{(x+3)(x-2)} = 0$$

$$\frac{x(x+1)}{(x+1)(x-2)} = 0$$

Num = 0
 $x=0,$

Den = 0
 $x=-1, 2$

$$25) \frac{(x+5)(x+3)}{(x+5)x-3} + \frac{x(x-3)}{x-5(x-3)} - \frac{x+5(x-5)}{x-5(x-5)} = 0$$

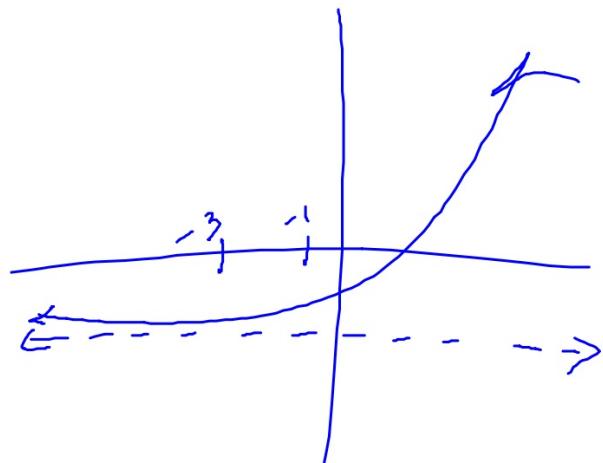
$$\frac{x^2 - 2x - 15 + x^2 - 3x - (x^2 + 2x - 15)}{(x-3)(x-5)} = 0$$

$$x(x-7) \frac{x^2 - 7x}{(x-3)(x-5)} = 0$$

$$x = \underline{0, 7}$$

2b)

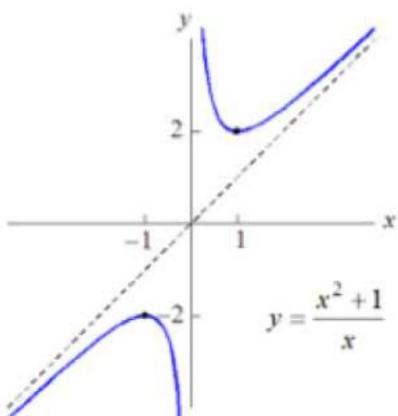
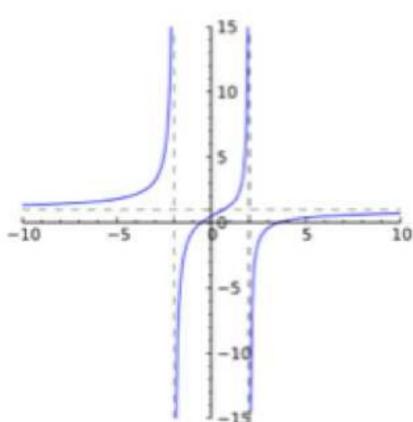
$$f(x) = 3^{x-1} - 1 \quad [-3, -1]$$



$$\sqrt{x+2}$$

$$[2, \infty)$$

5.3 Graphs of Rational Functions



When Sketching Rational Functions You Must Find:

- x-intercept(s)
- y-intercept
- asymptotes (HA, VA, SA)
- holes

x-intercepts

ex: Find the x-intercepts, if any.

(zeros)

$$f(x) = \frac{x^2 - x - 12}{x^2 - 2x - 8}$$

$$0 = \frac{x^2 - x - 12}{x^2 - 2x - 8}$$

$$0 = \frac{(x-4)(x+3)}{(x-4)(x+2)}$$

$$x = -3$$

*Factor,
Simplify,
then look
at the
numerator*

y-intercept

ex: Find the y-intercept, if any.

$$f(x) = \frac{x^2 - x - 12}{x^2 - 2x - 8}$$

$$f(0) = \frac{12}{8} = \frac{3}{2}$$

Finding Horizontal Asymptotes

(HA)

To find the horizontal asymptote compare the degree of the numerator and denominator. Three cases arise:

Case	Degree Numerator	$<$	Degree Denominator	Asymptote $y = 0$
2	Numerator	$>$	Denominator	none
3	Numerator	\equiv	Denominator	$y = \frac{a}{b}$

Rational functions can have at most ONE HA

Remembering Horizontal Asymptotes

BOBO	BOTN	EATSDC
Bigger on bottom O	Bigger on top none	exponents are the same divide coefficients

Horizontal Asymptotes

ex: Find the HA, if any.

a) $y = \frac{16x+1}{4x^2-2}$ $y = \textcircled{0}$

b) $y = \frac{\cancel{a}16x^2+1}{\cancel{b}4x^2-2}$ $y = 4$

c) $y = \frac{16x^4+1}{4x^2-2}$ none

Vertical
Finding ~~Horizontal~~ Asymptotes

To find vertical asymptotes:

1. Simplify.
2. Set the simplified denominator = 0.

Rational functions can have more than one VA

Vertical Asymptotes

ex: Find the VA, if any.

a) $y = \frac{x^2 - 4x - 5}{x^2 - 1}$

$$y = \frac{(x-5)(x+1)}{(x-1)(x+1)}$$
$$y = \frac{x-5}{x-1}$$

$$x-1 = 0$$

$$x = 1$$

VA

ex: Find the VA, if any.

b) $f(x) = \frac{3}{(x+8)^2}$

$$(x+8)^2 = 0$$
$$x = -8$$

c) $f(x) = \frac{x+1}{x^2+1}$

$$x^2 + 1 = 0$$

None

~~$x = \pm i$~~

d) $y = \frac{x-7}{x^3-8} =$

$$\frac{x-7}{(x-2)(x^2+2x+4)}$$

$$x = 2$$

Finding Slant (Oblique) Asymptotes

*A rational function has a slant asymptote when the degree of the numerator is EXACTLY one greater than the degree of the denominator AND the denominator is NOT a factor of the numerator.

To find slant asymptotes: ***Simplify first***

1. Divide. There MUST be a remainder.
2. Ignore the remainder.

Rational functions can at most one SA

ex: Is it possible for a rational function to have both a slant and horizontal asymptote? Explain.

No. Because in order to have a SA, the degree of the numerator must be one larger than the denominator.

For an HA, you will have either a BOBO or an EATSDC.

ex: Find the SA, if any.

a) $y = \frac{x^2 - 2x + 3}{x + 3}$

$$\begin{array}{r} -3 \\ \hline 1 & -2 & 3 \\ & -3 & /5 \\ \hline & 1 & -5 & \cancel{8} \end{array}$$

$$y = x - 5$$

ex: Find the SA, if any.

b) $f(x) = \frac{x^2 - 3x + 7}{x - 2}$

$$\begin{array}{r} 2 \\ \sqrt[3]{1 \quad -3 \quad 7} \\ \quad \quad 2 \quad -2 \\ \hline \quad \quad \quad -1 \quad \cancel{5} \end{array}$$

$$\boxed{y = x - 1}$$

ex: Find the SA, if any.

c) $f(x) = \frac{x+5}{x^2 + 4x + 8}$

none

ex: Find the SA, if any.

d) $y = \frac{x^2 - 9}{x + 3}$

$$y = \frac{(x+3)(x-3)}{x+3}$$

$$\begin{array}{r} -3 \\ \sqrt[3]{1 \quad 0 \quad -9} \\ \quad -3 \quad 9 \\ \hline \quad 1 \quad -3 \quad 0 \end{array}$$

$$y = x - 3$$

None!

Finding Holes

To find holes:

1. Factor completely.
2. If the numerator and denominator share a common factor a hole exists.
3. The hole exists at the zero of the common factor.
4. To find the y-value, plug in x into the SIMPLIFIED version.

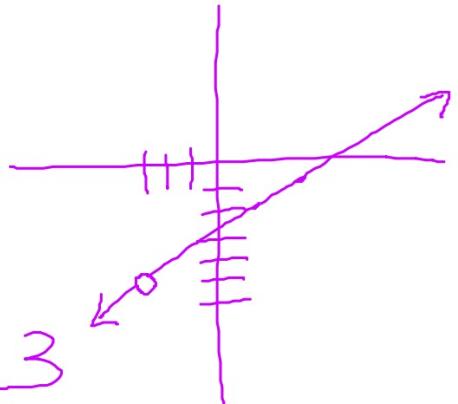
Rational functions can have more than one hole

ex: Find all holes, if any.

a) $y = \frac{x^2 - 9}{x + 3}$

$$y = \frac{(x-3)(x+3)}{(x+3)} = x - 3$$

Hole @ $(-3, -6)$



$$\boxed{y = x - 3}$$

ex: Find all holes, if any.

b) $y = \frac{x^2 + 4x - 5}{x^3 - 1}$

$$y = \frac{(x+5)(x-1)}{(x-1)(x^2+x+1)}$$

Hole @ (1, 2)

ex: Find the x-intercepts, y-intercept, asymptotes and holes, if any.

$$f(x) = \frac{5x^2 - 9x - 18}{x^2 - 3x}$$