

4.6 Solving Log & Exponential Equations - Mixed Practice & Finding Inverses

4.7 Exponential Word Problems - Compound Interest



Solve. Round to three decimal places if necessary.

If $f(x) = \log(x + 3)$, find $f^{-1}(2)$

$$13.) \log_6 x + \log_6 (x+6) = 1$$

$$\log_6 (x^2 + x) = 1$$

$$x^2 + x = 6$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2)$$

$$\cancel{x=3}, 2$$

2³

$$8.) \log_4 \left(\frac{2x+2}{x-2} \right) = 1$$

$$\frac{2x+2}{x-2} = 4$$

$$2x+2 = 4x-8$$

$$10 = 2x$$

$$5 = x$$

$$2.) \log_{\sqrt{3}} x = 6$$

$$x = (\sqrt{3})^6$$

$$x = (3^{1/2})^6$$

$$x = 27$$

$$14.) \log_3(x^2-3) = \log_3 2x$$

$$x^2 - 3 = 2x$$

$$x^2 - 2x - 3 = 0$$

$$6.) \log_{10}(x-7) = \frac{3}{10}$$

$$x-7 = 1000$$

$$11.) \quad \ln(-x) = 18$$

$$e^{\ln(-x)} = e^{18}$$

$$-x = e^{18}$$

$$x = -e^{18}$$

ex: Find the inverse.

a) $f(x) = 2 \cdot 3^{x-1} + 1$

$$X = 2 \cdot 3^{y-1} + 1$$

$$\frac{X-1}{2} = \frac{2 \cdot 3^{y-1}}{2}$$

$$\log_3 \frac{X-1}{2} = \log_3 3^{y-1}$$

$$\log_3 \frac{X-1}{2} = y-1$$

$$f^{-1}(x) = \log_3 \frac{x-1}{2} + 1$$

ex: Find the inverse.

$$b) g(x) = 2\log_4(x+5) - 4$$

$$x = 2\log_4(y+5) - 4$$

$$4^{\frac{x+4}{2}} = \log_4(y+5)$$

$$4^{\frac{x+4}{2}} = y+5$$

$$4^{\frac{x+4}{2}} - 5 = g^{-1}(x)$$

$$\frac{x+4}{2} = \frac{1}{2}x + 2$$

$x+5=0$
 $x=-5$
VA

HA
 $y=5$

Two Types

1. Compound Interest
2. Growth/ Decay Models

Compound Interest

For interest compounded n times per year:

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

Where:

- A *accumulated amount*
- P *Principal amount (starting amount)*
- r *interest rate (as a decimal)*
- n *number of times the money is compounded*
- t *time*

	n
Annually	1
Quarterly	4
Monthly	12
Weekly	52
Daily	365
Semiannually	2
Bimonthly	

Compound Interest

For interest compounded continuously:

$$A = Pe^{rt}$$

Where:

- A *Accumulated amount*
- P *Principal amount (starting amount)*
- r *Rate (as a decimal)*
- t *time*

ex 1: Find the total value of a \$7,300 investment it is invested at 7% annual interest compounded semiannually for 3 years.

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

Semiannually :

$$A = 7300 \left(1 + \frac{.07}{2}\right)^6 = \$8973.56$$

daily : $A = 7300 \left(1 + \frac{.07}{365}\right)^{1095} = \9005.67

ex 2: Find the total value of a \$7,300 investment it is invested at 7% annual interest compounded continuously for 3 years.

$$A = Pe^{rt}$$

$$A = 7300 \cdot e^{.07 \cdot 3}$$

$$= 7300e^{.21}$$

$$= \$9005.85$$

ex 3: Find the total value of a \$1.17 investment if it is invested at 9% annual interest compounded daily since 1927.

$$A = 1.17 \left(1 + \frac{.09}{365} \right)^{(365 \cdot 91)}$$

$$A = 1.17 \left(1 + \frac{.09}{365} \right)^{33215}$$

$$A = \$4213.27$$

ex 3: ABC Bank is offering to double your money! They say that if you invest with them at 6% interest compounded continuously they will double your money. If you invest \$1500 in the account, how long will it take to double your money.

$$A = Pe^{rt}$$

$$3 = 2^x$$

$$3000 = 1500e^{.06t}$$

$$\ln 2 = \ln e^{.06t}$$

$$\ln 2 = .06t (\ln e)$$

$$\frac{\ln 2}{.06} = t = 11.55 \text{ years}$$

ex 4: ABC Bank is offering to double your money! They say that if you invest with them at 6% interest compounded continuously they will ~~double~~ your money. If you invest \$1500 in the account, how long will it take to double your money.

↓
triple

$$\ln 3 = \ln e^{.06t}$$

$$\frac{\ln 3}{.06} = \frac{.06t \cdot \ln e}{.06}$$

$$18.31 = t$$

years

ex 5: An investment of \$7,000 becomes \$10,000 when invested for 5 years in a bank that compounds interest quarterly. What interest rate does the bank use?

$$10,000 = 7,000 \left(1 + \frac{r}{4}\right)^{20}$$
$$\sqrt[20]{\frac{10}{7}} = \sqrt[20]{\left(1 + \frac{r}{4}\right)^{20}}$$

$$1.0179937 = 1 + \frac{r}{4}$$

$$0.01974 = r$$

7.2%