4.6 Solving Exponential & Logarithmic Equations

Logic: another thing that penguins aren't very good at.

*See printout.

$$2^x = 12$$

Method #1

$$2^x = 12$$

$$\log_2 12 = x$$

$$\frac{\log 12}{\log 2} = x$$

or $\frac{\ln 12}{\ln 2}$

3.585

Method #2

$$2^x = 12$$

$$\log 2^x = \log 12$$

$$\frac{x \log 2}{\log 2} = \frac{\log 12}{\log}$$

$$x = \frac{\log 12}{\log 2}$$

3 Types of Exponential Equations:

1. $a^x = b$, where a and b are integral powers of the same number

$$\text{ex: } 27^x = 9$$

2. $a^x = b$, where a and b are NOT integral powers of the same number

$$\text{ex: } 3^x = 5$$

3. quadratic form

$$\text{ex: } 3^{2x} + 3^x - 6 = 0$$

Type 2

ex: Solve.

e) $3^x = 5$

$\log_3 5 = x$

$\frac{\log 5}{\log 3} = x$

$1.465 = x$

ex: Solve.

f) $e^{x+1} = 10$

$\log_e 10 = x + 1$

$\ln 10 = x + 1$

$\ln 10 - 1 = x$

$1.303 = x$

$\frac{\ln 10}{\ln e}$ ↑ 1

$\ln e^{x+1} = \ln 10$

$(x+1)\cancel{\ln e} = \ln 10$

$x + 1 = \ln 10$

ex: Solve.

g) $2 \cdot 10^{x-3} - 3 = 37$

$10^{x-3} = 20$

$\log 20 = x - 3$

$3 + \log 20 = x \quad x = 4.301$

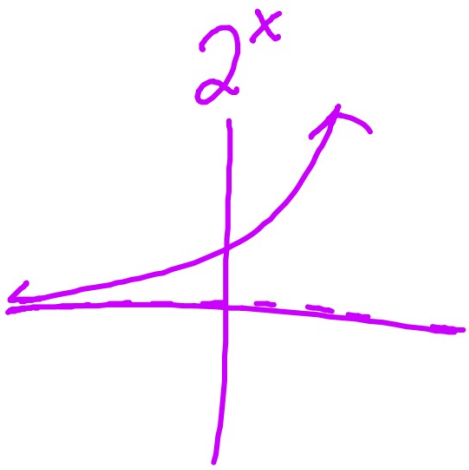
ex: Solve.

h) $2 - 5^{x-2} = 3$

$-5^{x-2} = 1$

$5^{x-2} = -1$

$\emptyset$ $\log_5(-1) = x-2$



ex: Solve.

i) $5^{x-1} = 3^{x+2}$

$\log 5^{x-1} = \log 3^{x+2}$

$(x-1)\log 5 = (x+2)\log 3$

$x\log 5 - \log 5 = x\log 3 + 2\log 3$

$x\log 5 - x\log 3 = \log 5 + 2\log 3$

$\frac{x(\log 5 - \log 3)}{\log 5 - \log 3} = \frac{\log 5 + \log 9}{\log 5 - \log 3} = \frac{\log 45}{\log \frac{5}{3}}$

7.452

$$4^{5-x} = 3^{x+1}$$

$$4^{5-x} = 3^{x+1}$$

$$\log 4^{5-x} = \log 3^{x+1}$$

$$(5-x)\log 4 = (x+1)\log 3$$

$$5\log 4 - x\log 4 = x\log 3 + \log 3$$

$$5\log 4 - \log 3 = x\log 3 + x\log 4$$

$$\log 1024 - \log 3 = x(\cancel{\log 3 + \log 4})$$

$$\frac{\log 1024 - \log 3}{\log 3 + \log 4} \rightarrow \frac{\log \frac{1024}{3}}{\log 12} = 2.347$$

Type 3

ex: Solve.

j) $2^{2x} - 5 \cdot 2^{x} - 24 = 0$

$x^2 - 5x - 24$

$(x-8)(x+3)$

$(2^x - 8)(2^x + 3) = 0$

$2^x - 8 = 0$	$2^x + 3 = 0$
$2^x = 8$	$2^x = -3$
$x = 3$	$\varnothing$

$x = 3$

ex: Solve.

k) $4^{2x} - 7 \cdot 4^{x} + 12 = 0$

$(4^x - 4)(4^x - 3) = 0$

$4^x - 4 = 0$ | $4^x - 3 = 0$

$4^x = 4$ | $4^x = 3$

$x = 1$ | $\log_4 3 = x = .792$

MIXED PRACTICE

ex: Solve.

(1) a) $5 \cdot 2^x - 3 = 157$

$x = 5$

(3) b) $3^{2x} - 4 \cdot 3^x - 12 = 0$

$x = 1.631$

(1) c) $8^{x+1} = 4^{x-3}$ $x = -9$

(1) d) $\dfrac{81^{3-x}}{3^{x+1}} = \left(\dfrac{1}{3}\right)^{6x-5}$

(2) e) $7^{x+10} = 3^{2x-1}$ $x = 81.801$

$$\frac{3^{4(3-x)}}{3^{x+1}} = 3^{-1(6x-5)}$$

$12 - 4x - (x+1) = -6x + 5$

$12 - 5x - 1 = -6x + 5$

$x = -6$

MIXED PRACTICE

ex: Solve.

1

a) $5 \cdot 2^x - 3 = 157$ $x = 5$

3

b) $3^{2x} - 4 \cdot 3^x - 12 = 0$ $x = 1.631$

1

c) $8^{x+1} = 4^{x-3}$ $x = -9$

1

d) $\dfrac{81^{3-x}}{3^{x+1}} = \left(\dfrac{1}{3}\right)^{6x-5}$ $x = -6$

2

e) $7^{x+10} = 3^{2x-1}$ $x = 81.801$

a) $5 \cdot 2^{x} - 3 = 157$

b) $2 \cdot 3^{2x} - 4 \cdot 2^{x} - 12 = 0$

c) $8^{x+1} = 4^{x-3}$

d) $\dfrac{81^{3-x}}{3^{x+1}} = \left(\dfrac{1}{3}\right)^{6x-5}$

e) $7^{x+10} = 3^{2x-1}$