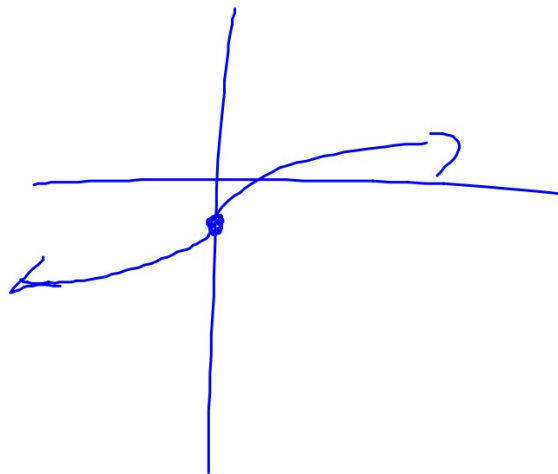


$$19.) y = \frac{3}{4} \sqrt[3]{X} - 1$$

$$\frac{3}{4} X^{1/3}$$

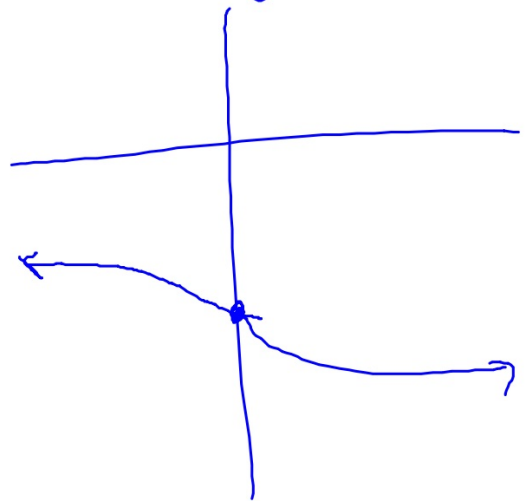
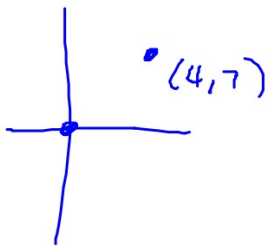
X	y
-8	$-\frac{3}{2} - 1 = -2\frac{1}{2}$
-1	$-\frac{3}{4} - 1 = -1\frac{3}{4}$
0	-1
1	$\frac{3}{4} - 1 = -1/4$
8	$\frac{3}{2}$



$$23.) \quad g(x) = -\frac{1}{3}\sqrt[3]{x} - 6$$

Key: $(0, -6)$

$$y = -(\underbrace{x-4}_{\text{red circle}})^{1/3} + 7$$



$$y = 5 - \sqrt[3]{5x+10}$$

$$y = -\sqrt[3]{5x+10} + 5$$

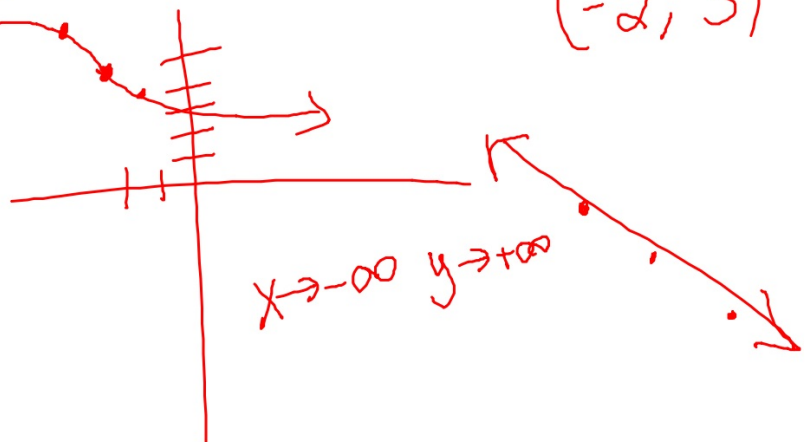
$$y = -\sqrt[3]{5(x+2)} + 5$$

$$5x+10=0$$

$$x = -2$$

$$(-2, 5)$$

x	y
-11/5	6
-2	5
-9/5	4



$$f(g(x)) = \frac{1}{8}x^3 - 3$$

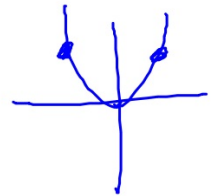
$$\left(\frac{1}{8}x^3 - 3\right)^{-1}(x)$$

$$y = \frac{1}{8}x^3 - 3$$

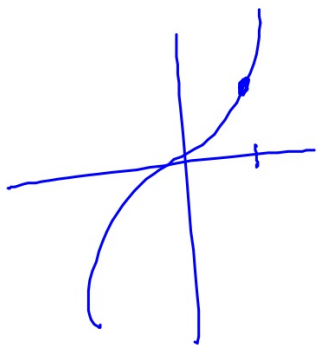
$$x = \frac{1}{8}y^3 - 3$$

$$20.) \quad 10 = x^3 + x \quad h^{-1}(10) = 2$$

$$0 = x^3 + x - 10$$



$\pm 1, \pm 2, \pm 5, \pm 10$



$$\begin{array}{r} \textcircled{2} \quad \begin{array}{rrrr} 1 & 0 & 1 & -10 \\ & 2 & 4 & 10 \\ \hline 1 & 2 & 5 & 0 \end{array} \\ x^2 + 2x + 5 = 0 \end{array}$$

$$6.) \quad h(x) = \frac{x+1}{x-1}$$

$$\frac{x}{1} = \frac{y+1}{y-1}$$

$$y^{-1} = \frac{x+1}{x-1}$$

$$xy - x = y + 1 \quad \rightarrow$$

$$xy - y = x + 1 \quad \leftarrow$$

$$y(x-1) = x+1$$

3.6 Radical Equations

Square Root Equations - 1 Root

ex: Solve. (REAL SOLUTIONS ONLY)

a) $\sqrt{x-5} + 2 = 7$

$$(\sqrt{x-5})^2 = (5)^2$$
$$x-5 = 25$$
$$\boxed{x=30}$$

Steps:

1) Isolate the radical

2) Solve (eliminate the radical)

3) Check for extraneous

Check:

$$\sqrt{x-5} + 2 = 7$$
$$\sqrt{30-5} + 2 = 7$$
$$\sqrt{25} + 2 = 7$$
$$7 = 7 \checkmark$$

ex: Solve.

b) $3\sqrt{x+2} - 4 = -10$

$$\sqrt{x+2} = -2$$

$$x+2 = 4$$

$$x = 2$$

check:

$$\sqrt{x+2} = -2$$

$$\sqrt{4} \neq -2$$

No Solution or \emptyset

ex: Solve.

$$c) 1 - 2\sqrt{x^2 - 5x + 15} = -5$$

$$\sqrt{x^2 - 5x + 15} = 3$$

$$x^2 - 5x + 15 = 9$$

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3) = 0$$

$$x = 2, 3$$

Square Root Equations - More Than 1 Root

ex: Solve.

$$d) (\sqrt{5x-6})^2 = (3\sqrt{x-1})^2$$

$$5x-6 = 9(x-1)$$

$$5x-6 = 9x-9$$

$$3 = 4x$$

$$\frac{3}{4} = x$$

$$\sqrt{5 \cdot \frac{3}{4} - 6} \neq 3\sqrt{\frac{3}{4} - 1}$$

No solution

ex: Solve.

$$e) \left(\sqrt{x-3} \right)^2 = \left(\sqrt{x+4} - 1 \right)^2$$

$$x-3 = (\sqrt{x+4} - 1)(\sqrt{x+4} - 1)$$

$$x-3 = x+4 - \sqrt{x+4} - \sqrt{x+4} + 1$$

$$x-3 = x+4 - 2\sqrt{x+4} + 1$$

$$2\sqrt{x+4} = 8$$

$$\sqrt{x+4} = 4 \Rightarrow x+4 = 16$$

$$x = 12 \checkmark$$

Steps

1) Put the radicals on opposite sides of the equation

2) Square both sides

3) Isolate the radical

4) Square again

5) Solve, check for extraneous

$$\begin{aligned} \sqrt{x-3} &= \sqrt{x+4} - 1 \\ \sqrt{9} &= \sqrt{16} - 1 \end{aligned}$$

ex: Solve.

f) $\sqrt{x} + \sqrt{x+5} = 5$

$$(\sqrt{x})^2 = (5 - \sqrt{x+5})^2$$

$$x = 25 - 10\sqrt{x+5} + x + 5$$

$$10\sqrt{x+5} = 30$$

$$\sqrt{x+5} = 3$$

$$x+5 = 9$$
$$x = 4$$

$$(5 - \sqrt{x+5})(5 - \sqrt{x+5})$$
$$-5\sqrt{x+5} - 5\sqrt{x+5}$$

Nth Root Equations

*Check for extraneous solutions
with even roots. Why?
The domain is not all reals.*

ex: Solve.

$$\text{h) } \sqrt[3]{2x+1} - 4 = 1$$

$$\left(\sqrt[3]{2x+1}\right)^3 = (5)^3$$

$$2x+1 = 125$$

$$x = 62$$

ex: Solve.

$$i) \sqrt[4]{5x^2 - 4} = x^4$$

$$5x^2 - 4 = x^4$$

$$x^4 - 5x^2 + 4 = 0$$

$$(x^2 - 4)(x^2 - 1) = 0$$

$$\pm 2, \pm 1$$

$$x = 1, 2$$

ex: Solve.

$$i) \sqrt{x} = \sqrt[4]{x}$$

$$(x^{1/2})^4 = (x^{1/4})^4$$

$$x^2 = x$$

$$x^2 - x = 0$$

$$x(x-1) = 0$$

$$x = 0, 1$$

ex: Solve.

l) $x^2 = 4$

$$x = \pm 2$$

$$(x^2)^{1/2} = (4)^{1/2}$$

$$x = \pm \sqrt{4}$$

m) $x^3 = -8$

$$(x^3)^{1/3} = (-8)^{1/3}$$

$$x = \sqrt[3]{-8} = -2$$

Rational Exponent Equations

ex: Solve.

$$n) (x^{3/4})^{4/3} = (8)^{4/3}$$

$$x^1 = 16$$

odd root
no \pm necessary

$$o) (x^{2/3})^{3/2} = (9)^{3/2}$$

$$x = \pm 9^{3/2}$$

$$x = \pm 27$$

even roots!
 \pm

If you take the even root of both sides, it is just like a square root so \pm will be necessary

ex: Solve.

n) $3x^{3/2} - 2 = 373$

$$(x^{3/2})^{2/3} = (125)^{2/3}$$

$$x = 25$$

o) $4x^{2/3} + 5 = 41$

$$(x^{2/3})^{3/2} = (9)^{3/2}$$

$$x = \pm 27$$

ex: Solve.

$$p) 2(x+4)^{2/3} + 1 = 19$$

$$\left((x+4)^{2/3} \right)^{3/2} = (9)^{3/2}$$

$$-4 + 27$$

$$x+4 = \pm 27$$

$$-4 - 27$$

$$x = -4 \pm 27$$

$$x = 23, -31$$

ex: Solve.

$$q) 5 - 3(2x - 1)^{2/3} = 32$$

$$\left((2x - 1)^{2/3} \right)^{3/2} = (-9)^{3/2}$$

sq. root of
negative

No solution

ex: Solve.

$$r) \frac{1}{2}(x+3)^{5/2} - 4 = 12$$

$$\left((x+3)^{5/2} \right)^{2/5} = (32)^{2/5}$$

$$x+3 = 4$$

$$x = 1$$

ex: Solve.

s) $3(x^2 - 5x - 5)^{7/6} - 4 = -1$ check (even root)

$$(x^2 - 5x - 5)^{7/6} = 1$$

$$x^2 - 5x - 5 = 1$$

$$x^2 - 5x - 6 = 0$$

$$(x - 6)(x + 1) = 0$$

$$(x - 1)$$

Graph square roots and cube roots with D and R

Inverse functions

verifying that functions are inverses

finding an inverse function

evaluating an inverse function (example $f^{-1}(2)$)

determine if a function has an inverse function

(HLT and VLT)

Solving radical equations

Solving fractional exponent equations

ex: Solve.

t) $x^{3/2} = x$

REVIEW

ex: Sketch and state the domain and range in set notation.

$$y = -2\sqrt{5-x}$$

REVIEW

ex: Sketch and state the domain and range in set notation.

$$y = 3\sqrt[3]{4x+1} - 2$$

REVIEW

ex: Solve.

$$\sqrt{5x+6} + 3 = \sqrt{3x+3} + 4$$

