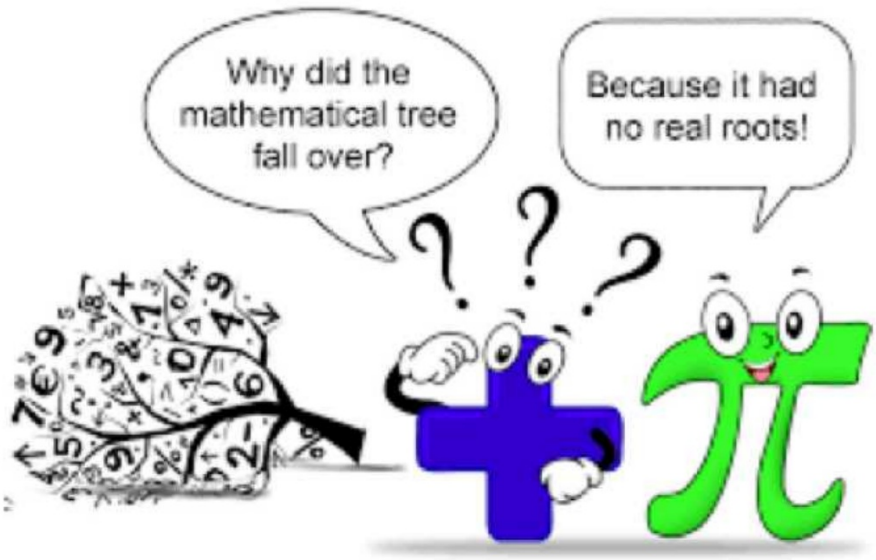
# 2.7 Finding All Zeros Cont.
# Analyzing Polynomial Functions




*See printout.

HW:

## Polynomial Degrees and Number of Turning Points

| Polynomial Type | Degree | Maximum Number of Turning Points |
|---|---|---|
| Constant | 0 | 0 |
| Linear | 1 | 0 |
| Quadratic | 2 | 1 |
| Cubic | 3 | 2 |
| $n^{th}$ Degree Polynomial | $n$ | $n-1$ |

## Complex Conjugate & Irrational Conjugate Theorem

Imaginary and irrational roots always come in

*Conjugate pairs*.

ex: Write a polynomial function in standard form with integral coefficients and the given roots.

a) $-\frac{2}{5}, \; 3i, \; -3i$

$f(x) = (5x + 2)(x^2 + 9)$

$f(x) = 5x^3 + 2x^2 + 45x + 18$

ex: Write a polynomial function in standard form with integral coefficients and the given roots.

b) 0 multiplicity 5, $-\sqrt{3}$, $\sqrt{3}$

$f(x) = x^5(x^2 - 3)$

$f(x) = x^7 - 3x^5$

ex: Write a polynomial function in standard form with integral coefficients and the given roots.

b) 0 multiplicity 5, $1-\sqrt{3}$, $1+\sqrt{3}$

$f(x) = x^5(x^2 - 2x - 2)$

$f(x) = x^7 - 2x^6 - 2x^5$

Sum: $-(2)$

product: $(1-\sqrt{3})(1+\sqrt{3})$

$1 - 3$

$-2$

$x = \frac{2 \pm \sqrt{4-4(1)(-2)}}{2(1)} = \frac{2 \pm 2\sqrt{3}}{2}$

*Sum and Product Rule*

$$f(x) = x^2 + bx + c$$

| $-3, 6$ | $-5, -1$ |
| --- | --- |
| Sum: $-(-3+6) = b$ | Sum: $-(-5+-1) = 6$ |
| $-3 = b$ | product: $(-5)(-1) = 5$ |
| product: $(-3)(6) = c$ | $f(x) = x^2 + 6x + 5$ |
| $-18 = c$ | |
| $f(x) = x^2 - 3x - 18$ | |

$-\frac{1}{2}, 2$

Sum: $-\left(-\frac{1}{2}+2\right)$

$-\frac{3}{2}$

product: $-1$

$2\left(x^2-\frac{3}{2}x-1=0\right)$

$2x^2-3x-2=0$

ex 1) Determine the degree and state the maximum number of turning points.

a) $f(x) = 2x^3 + 5x^2 - 9$

3 / 2

b) $f(x) = 9 - 6x^2$

2 / 1

ex 1) Determine the degree and state the maximum number of turning points.

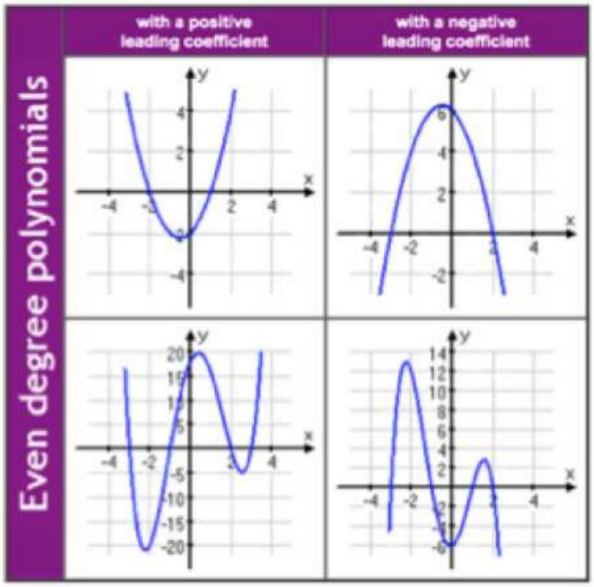
c) $f(x)=(x-2)^5(x+3)^6$

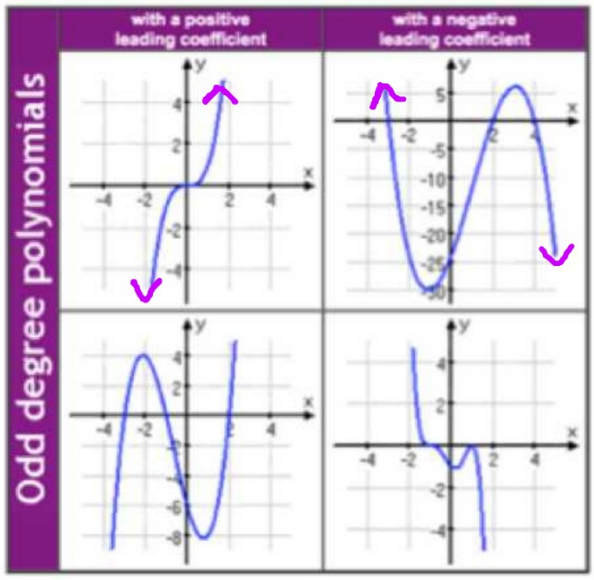
11 / 10

d) $f(x)=5(1-x^3)^7$

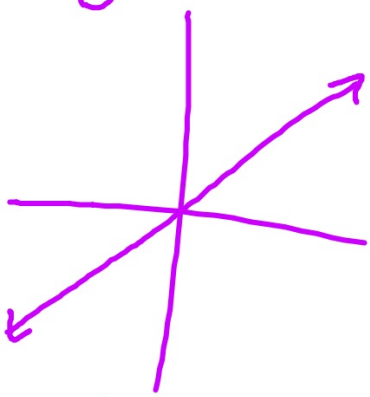
14 / 13

# End Behavior

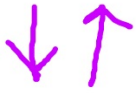


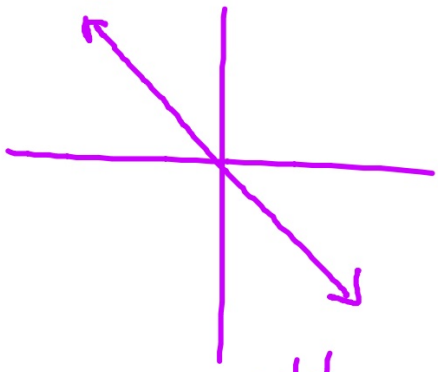
$y = x$



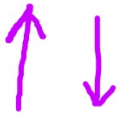
odd
(+)



$y = -x$



odd
(−)



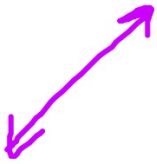
## Stating End Behavior

$x \to -\infty, \quad y \to$ _____

$x \to \infty, \quad y \to$ _____

ex 2) Determine the end behavior of each polynomial.

a) $f(x) = 2x^3 + 5x^2 - 9$



$x \to -\infty \quad y \to -\infty$

$x \to \infty \quad y \to \infty$

b) $f(x) = 9 - 6x^2$

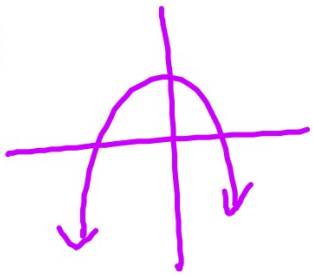
even

(−)

$x \to -\infty \quad y \to -\infty$

$x \to \infty \quad y \to -\infty$



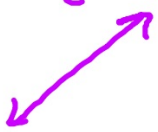


ex 2) Determine the end behavior of each polynomial.

c) $f(x)=(x-2)^5(x+3)^6$

odd
(+)

$x \to -\infty \quad y \to -\infty$
$x \to \infty \quad y \to \infty$



d) $f(x)=5(1-x^3)^7$

even
(-)

$x \to -\infty \quad y \to -\infty$
$x \to \infty \quad y \to -\infty$

$f(x) = 7x^6 - 2x^4 - x + 1$

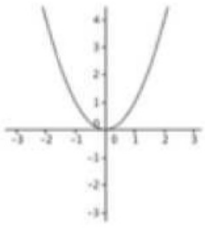
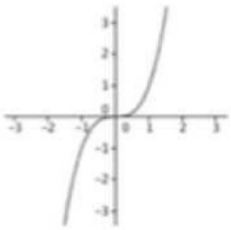
even

(+)

$x \to -\infty \quad y \to \infty$

$x \to \infty \quad y \to \infty$

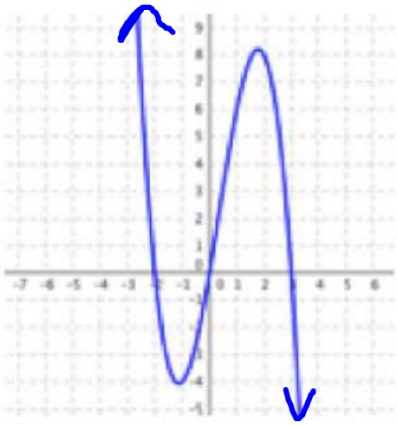
# Bouncing and Crossing Zeros

| | |
|---|---|
| In the graph below the graph "bounces" off the x-axis at x=0.  | In the graph below the graph "crosses" the x-axis at x=0.  |

ex 3) Using the graph of the polynomial function,

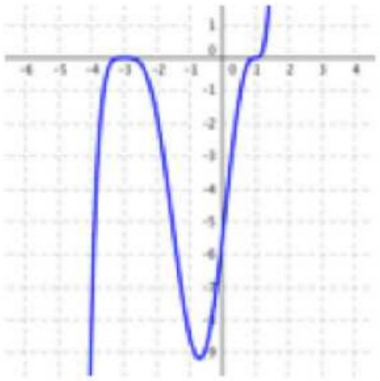
1. State the degree of the polynomial.
2. Find the zeros. State the multiplicity if greater than 1.
3. State the end behavior.
4. Determine whether the graph "crosses" the x-axis or "bounces" off the x-axis at each zero.

a) $f(x) = -x(x+2)(x-3)$



1. 3
2. $x = 0, -2, 3$
3. $x \to -\infty \quad y \to \infty$
   $x \to \infty \quad y \to -\infty$
4. $x=-2$ cross $\quad x=0$ cross $\quad x=3$ cross

b) $f(x)=\frac{1}{15}(x+3)^4(x-1)^3$



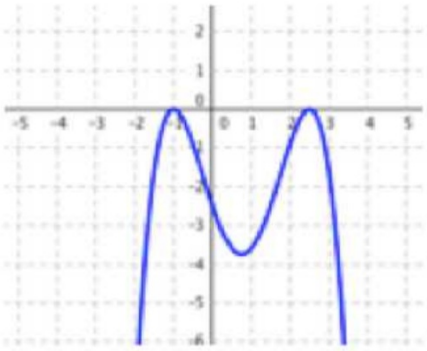
1. 7

2. $x=-3$ mult of 4 $\quad x=1$ mult. of 3

3. $x\to-\infty$ $y\to-\infty$
   $x\to\infty$ $y\to\infty$

4. $-3$ bounce $\quad 1$ cross

c) $f(x) = -\frac{1}{10}(2x-5)^2(x+1)^2$



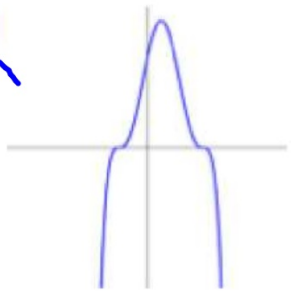
1. 4
2. $x = \frac{5}{2}$ m. of 2 $x = -1$ m. of 2
3. $x \to -\infty$ $y \to -\infty$
   $x \to \infty$ $y \to -\infty$
4. all bounce

- A graph "crosses" the x-axis at a zero if the multiplicity of that zero is ___odd___.

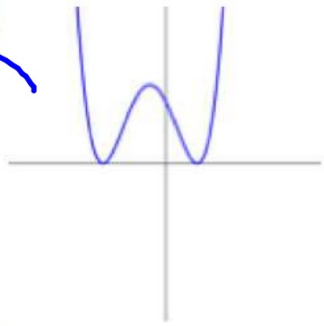
- A graph "bounces" off the x-axis at a zero if the multiplicity of that zero is ___even___.

ex 4) Which of the following graphs could represent the polynomial $f(x) = a(x-b)^2(x-c)^3$ ?

a)

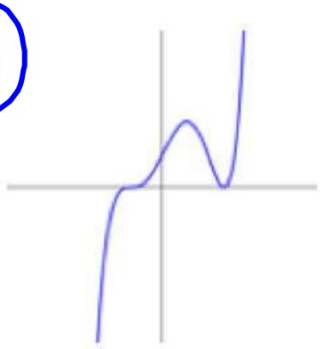


c)



b)

d)



Review

ex: Determine the number of imaginary zeros with the given degree and graph.

Degree: 4