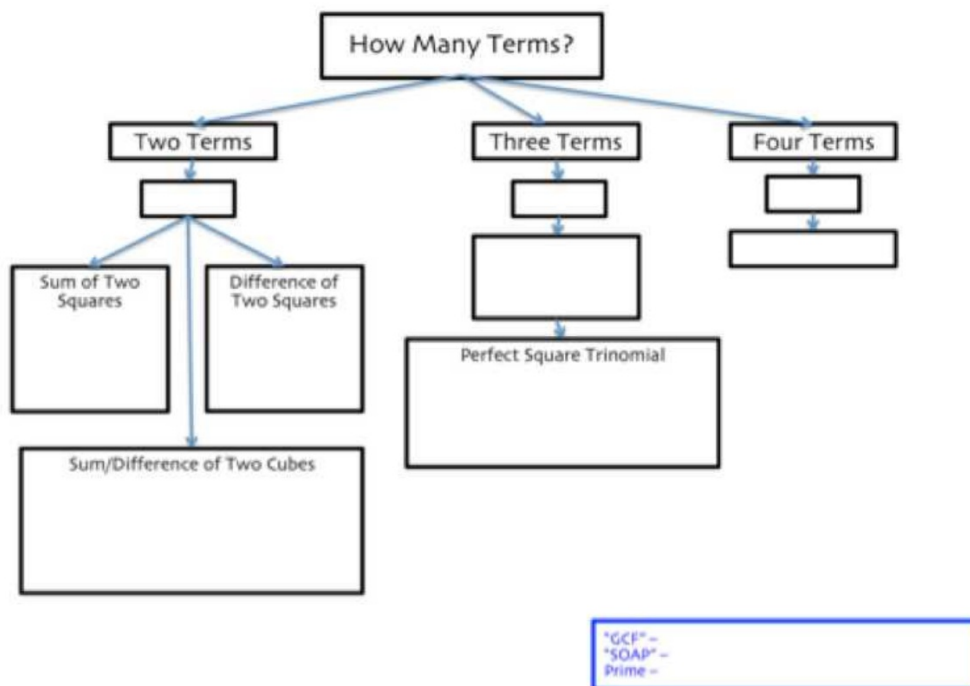


Factoring Bootcamp 2

2.4 Factor and Solve Polynomial Equations

*Grab your factoring flowchart!



2.4

Perfect Cubes

$$1^3 = 1$$

$$2^3 = 8$$

$$3^3 = 27$$

$$4^3 = 64$$

$$5^3 = 125$$


$$6^3 = 216$$

$$10^3 = 1000$$

Sum/Difference of Cubes

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 \neq (a+b)^3$$


Remembering the SIGNS in the Cubes Formula

SOAP

S - same sign

O - opposite sign

AP - always positive

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

SOAP

ex: Factor completely.

$$\text{a) } x^3 - 27 = (x - 3)(x^2 + 3x + 9)$$

$$a = x$$

$$b = 3$$

$$\text{b) } x^3 + 216 = (x + 6)(x^2 - 6x + 36)$$

$$a = x$$

$$b = 6$$

ex: Factor completely.

$$\text{c) } 8x^3 + 1 = (2x + 1)(4x^2 - 2x + 1)$$

$$a = 2x$$

$$b = 1$$

$$\text{d) } 7000 - 7x^3 = 7(1000 - x^3)$$

$$a = 10$$

$$b = x$$

$$= 7(10 - x)(\underline{100 + 10x + x^2})$$

Quadratic Form

$$ax^{2n} + bx^n + c$$

$$ax^{2n} + bx^n + c$$

$$x^2 + 4x + 3$$

$$x^4 + 4x^2 + 3$$

$$(x+3)(x+1)$$

$$(x^2+3)(x^2+1)$$

ex: Factor completely.

a) ~~$x^2 - 3x - 4$~~ $x^4 - x^2 - 12$

$$(x^2 - 4)(x^2 + 3)$$
$$(x+2)(x-2)(x^2+3)$$

b) $x^{10} - 3x^5 - 4$ $(x^5 - 4)(x^5 + 1)$

ex: Factor completely.

$$\text{g) } 2x^4 + 7x^2 + 6$$
$$(2x^2 + 3)(x^2 + 2)$$

$$\begin{array}{r} 2 \\ \hline 2 \end{array} \begin{array}{r} 1 \\ 3 \end{array}$$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\text{h) } 2x^{11} - 9x^6 + 10x$$

$$x(2x^{10} - 9x^5 + 10)$$

$$x(2x^5 - 5)(x^5 - 2)$$

ex: Factor completely.

$$k) x^5 - x^3 + 64x^2 - 64$$

$$x^3(x^2 - 1) + 64(x^2 - 1)$$

$$(x^3 + 64)(x^2 - 1)$$

$$(x + 4)(x^2 - 4x + 16)(x + 1)(x - 1)$$

$$a = x$$

$$b = 4$$

Theorem:

A polynomial equation with degree n has n solutions.

Vocabulary:

solutions/roots - answers to an equation

zeros - quantities that make a function equal to zero

ex: Solve by factoring.

$$b) 2x^4 + 7x^2 - 15 = 0$$

$$(2x^2 - 3)(x^2 + 5) = 0$$

$$2x^2 - 3 = 0$$

$$\sqrt{x^2} = \sqrt{\frac{3}{2}}$$

$$|x| = \frac{\sqrt{3}}{\sqrt{2}}$$

$$x = \pm \frac{\sqrt{6}}{2}$$

$$x^2 + 5 = 0$$

$$\sqrt{x^2} = \sqrt{-5}$$

$$|x| = \pm i\sqrt{5}$$

$$x = \pm i\sqrt{5}$$

ex: Solve by factoring.

$$c) 24x^4 + 3x = 0$$

$$3x(8x^3 + 1) = 0$$

$$3x(2x+1)(4x^2 - 2x + 1) = 0$$

$$\begin{array}{l} \downarrow \quad \downarrow \\ \textcircled{x=0} \quad \textcircled{x=-\frac{1}{2}} \end{array} \quad \downarrow \quad x = \frac{2 \pm \sqrt{-12}}{8} = \frac{2 \pm 2i\sqrt{3}}{8} = \frac{1}{4} \pm \frac{\sqrt{3}}{4}i$$

ex: Solve by factoring.

$$x^2(x-5) - 9(x-5)$$

d) $x^3 - 5x^2 - 9x + 45 = 0$

$$(x^2 - 9)(x - 5) = 0$$

$$(x+3)(x-3)(x-5) = 0$$

$$x = -3, 3, 5$$

ex: Solve by factoring.

e) $x^4 + 2x^2 + 1 = 0$

$$(x^2 + 1)(x^2 + 1) = 0$$

$$x^2 + 1 = 0$$

$$\sqrt{x^2} = \sqrt{-1}$$

$$x = \pm i, \text{ mult. of 2}$$

f.) $x^7 - 64x^5 = 0$

$$x^5(x - 8)(x + 8) = 0$$

$$x = 8, -8$$

$$0 \text{ (mult. of 5)}$$

ex: Solve by factoring.

$$g) -2x^7(x^2 - 2)^2(3x + 4) = 0$$

\downarrow
 $x = 0$, mult. of 7 $\rightarrow x = -\frac{4}{3}$

$$x^2 - 2 = 0$$

$$x = \pm\sqrt{2}, \text{ mult. of } 2$$

ex: Write a polynomial equation in standard form with integral coefficients and the given solutions.

a) 5, $-\frac{2}{3}$, 0

$$(x-5)(3x+2)x = 0$$

$$3x^3 - 13x^2 - 10x = 0$$

ex: Write a polynomial equation in standard form with integral coefficients and the given solutions.

b) $2i, -2i, 0$ multiplicity 2

$$(x - 2i)(x + 2i)x^2 = 0$$

$$(x^2 - 4i^2)x^2$$

$$(x^2 + 4)x^2 = 0$$

$$x^4 + 4x^2 = 0$$

$$x^2 + 16 = 0$$

↓

$$\pm 4i$$

$$x^2 - 16 = 0$$

↓

$$\pm 4$$

Review

ex: Evaluate using synthetic substitution.

$$f(x) = 5x^4 - x^3 + 7, \quad f(3) = ?$$

Review

ex: Perform the indicated operation.

$$(x^2 + x - 6) - (5x^2 - 2x + 3)$$