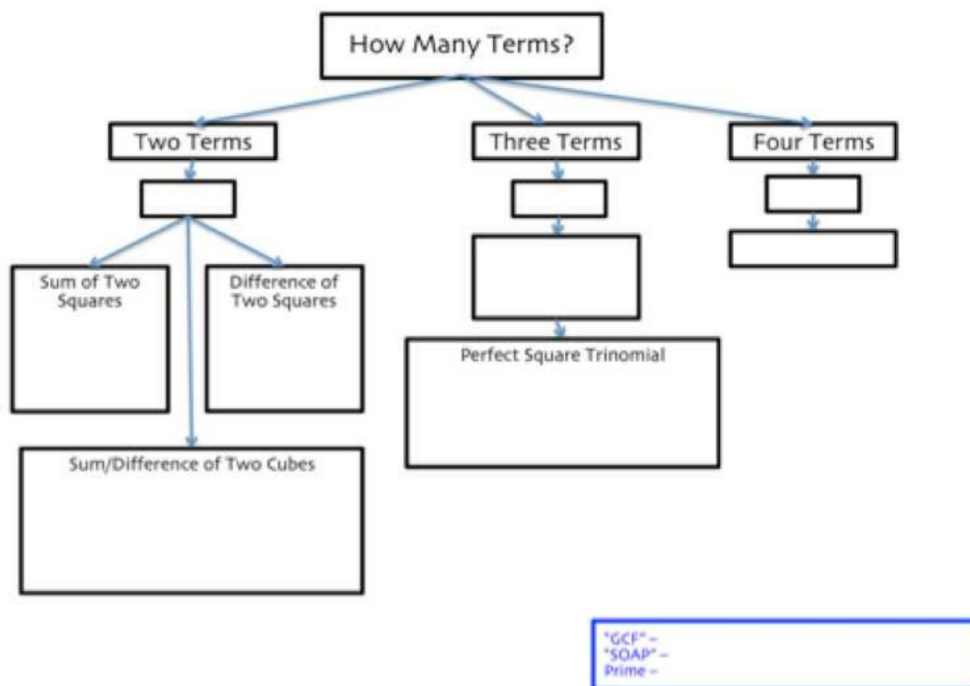


Factoring Bootcamp 2

2.4 Factor and Solve Polynomial Equations

*Grab your factoring flowchart!



REVIEW:

ex: Factor completely.

a) $x^2 - 9$ $(x+3)(x-3)$

b) $2x^2 - 3x - 14$ $(2x-7)(x+2)$

c) $9x^2 + 12x + 4$ $(3x+2)^2$

REVIEW:

ex: Factor completely.

d) $2x^2 + 162$

$$2(x^2 + 81)$$

e) $x^3 + 3x^2 - 2x - 6$

$$x^2(x+3) - 2(x+3)$$

$$(x^2 - 2)(x+3)$$

f) $x^3 - 3x^2 - 16x + 48$

$$x^2(x-3) - 16(x-3) = (x^2 - 16)(x-3)$$

$$= (x+4)(x-4)(x-3)$$

Perfect Cubes

$$1^3 = 1$$

$$2^3 = 8$$

$$3^3 = 27$$

$$4^3 = 64$$

$$5^3 = 125$$

$$6^3 = 216$$

$$10^3 = 1000$$

Sum/Difference of Cubes

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Remembering the SIGNS in the Cubes Formula

SOAP

S - same sign

O - opposite sign

AP - always positive

ex: Factor completely.

a) $x^3 - 27$ $(x - 3)(x^2 + 3x + 9)$
 $a = x$
 $b = 3$

b) $x^3 + 216$ $(x + 6)(x^2 - 6x + 36)$
 $a = x$
 $b = 6$

ex: Factor completely.

c) $8x^3 + 1$ $(2x + 1)(4x^2 - 2x + 1)$
 $a = 2x$
 $b = 1$

d) $7000 - 7x^3$
 $7(1000 - x^3) = 7(10 - x)(100 + 10x + x^2)$
 $a = 10$
 $b = x$

Quadratic Form

$$ax^{\underline{2n}} + bx^{\underline{n}} + c$$

$$x^2 + 7x^1 + 12$$

ex: Factor completely.

a) $x^2 - 3x - 4$
 $(x - 4)(x + 1)$

b) $x^{10} - 3x^5 - 4$ let $a = x^5$ $a^2 = x^{10}$
 $a^2 - 3a - 4$
 $(a - 4)(a + 1)$
 $(x^5 - 4)(x^5 + 1)$

ex: Factor completely.

c) $x^{\frac{2}{13}} - 3x^{\frac{1}{13}} - 4$

d) $x^{2n} - 3x^n - 4$

ex: Factor completely.

e) $x^4 - 3x^2 - 4$ $a = x^2$

$$\begin{aligned} & a^2 - 3a - 4 \\ & (a-4)(a+1) \\ & (x^2-4)(x^2+1) = (x+2)(x-2)(x^2+1) \end{aligned}$$

f) $x^6 - 3x^3 - 4$

$$(x^3-4)(x^3+1) = (x^3-4)(x+1)(x^2-x+1)$$

↑
diff of
2 cubes

ex: Factor completely.

g) $2x^4 + 7x^2 + 6$

$$(2x^2 + 3)(x^2 + 2)$$

h) $2x^{11} - 9x^6 + 10x$

$$x(2x^{10} - 9x^5 + 10)$$

$$x(2x^5 - 5)(x^5 - 2)$$

ex: Factor completely.

i) $8x^6 - 7x^3 - 1$

j) $16x^4 - 24x^2 + 9$

ex: Factor completely.

k) $x^5 - x^3 + 64x^2 - 64$

$$x^3(x^2 - 1) + 64(x^2 - 1)$$

$$(x^3 + 64)(x^2 - 1)$$

↑
sum of
2 cubes

↑
diff.
of
squares

$$(x+1)(x-1)(x+4)(x^2-4x+16)$$

Theorem:

A polynomial equation with degree n has n solutions.

Vocabulary:

solutions/roots - answers to an equation

zeros - quantities that make a function equal to zero

ex: Solve by factoring.

a) $x^2 - 8x + 15 = 0$

ex: Solve by factoring.

$$\text{b) } 2x^4 + 7x^2 - 15 = 0$$

$$(2x^2 - 3)(x^2 + 5) = 0$$

$$2x^2 - 3 = 0$$

$$\sqrt{x^2} = \sqrt{\frac{3}{2}} \sqrt{2}$$

$$x = \pm \frac{\sqrt{6}}{2}$$

$$x^2 + 5 = 0$$

$$\sqrt{x^2} = \sqrt{-5}$$

$$x = \pm i\sqrt{5}$$

ex: Solve by factoring.

$$c) 24x^4 + 3x = 0$$

$$3x(8x^3 + 1) = 0$$

$$3x(2x+1)(4x^2-2x+1) = 0$$

$$3x=0 \quad 2x+1=0 \quad 4x^2-2x+1=0$$

$$x=0$$

$$x = -\frac{1}{2}$$

$$x = \frac{2 \pm \sqrt{4 - 4(4)(1)}}{2(4)} = \frac{2 \pm \sqrt{-12}}{8}$$

$$= \frac{2 \pm 2i\sqrt{3}}{8} = \frac{1}{4} \pm \frac{i\sqrt{3}}{4}$$

ex: Solve by factoring.

$$d) x^3 - 5x^2 - 9x + 45 = 0$$

$$(x+3)(x-3)(x-5) = 0$$

$$x = -3, 3, 5$$

ex: Solve by factoring.

e) $x^4 + 2x^2 + 1 = 0$

$$(x^2 + 1)^2 = 0$$

$$x^2 + 1 = 0$$

$$\sqrt{x^2} = \sqrt{-1}$$

$$x = \pm i$$

$$x = i \text{ (mult. of 2)}$$

$$x = -i \text{ (mult. of 2)}$$

or

$$x = \pm i \text{ (mult. of 2)}$$

ex: Solve by factoring.

$$f) x^7 - 64x^5 = 0$$

$$x^5(x^2 - 64) = 0$$

$$x^5(x+8)(x-8) = 0$$

$$x^5 = 0 \quad x+8 = 0 \quad x-8 = 0$$

$$x = 0 \quad x = -8 \quad x = 8$$

m.o.s

ex: Solve by factoring.

$$g) -2x^7(x^2 - 2)^2(3x + 4) = 0$$

ex: Write a polynomial equation in standard form with integral coefficients and the given solutions.

a) 5, $-\frac{2}{3}$, 0

$$(x - 5)(3x + 2)x = 0$$

$$(3x^2 - 13x - 10)x = 0$$

$$\boxed{3x^3 - 13x^2 - 10x = 0}$$

ex: Write a polynomial equation in standard form with integral coefficients and the given solutions.

b) $2i, -2i, 0$ multiplicity 2

$$(x-2i)(x+2i)x^2 = 0$$

$$(x^2 - 4i^2)x^2 = 0$$

$$(x^2 + 4)x^2 = 0$$

$$x^4 + 4x^2 = 0$$

Review

ex: Evaluate using synthetic substitution.

$$f(x) = 5x^4 - x^3 + 7, \quad f(3) = ? \quad 385$$

$$\begin{array}{r|rrrrr} 3 & 5 & -1 & 0 & 0 & 7 \\ & \downarrow & 15 & 42 & 126 & 378 \\ \hline & 5 & 14 & 42 & 126 & 385 \end{array}$$

Review

ex: Perform the indicated operation.

$$(x^2 + x - 6) - (5x^2 - 2x + 3)$$